4. Adjustment by elements

The chapter on deformation analysis demands basic knowledge of the adjustment by elements. Therefore the basics will be reviewed briefly here. The aim of the adjustment is to obtain the optimal results from the observations. To do this one uses the method of least squares. In matrix format the least squares adjustment is formulated as follows.

$$\underset{n\times 1}{L} + \underset{n\times 1}{V} = \underset{n\times m}{A}\ \underset{m\times 1}{x} \qquad (4.1)$$

$$x = (A^tA)^{-1}A^tL \qquad (4.2)$$

$$V = Ax - L \qquad (4.3)$$

$$s_0 = \sqrt{\frac{V^tV}{f}} \qquad (4.4)$$

where L is the vector of observations, V the vector of residuals, A the design matrix of the system, x the sought unknowns, n the number of observations, m the number of unknowns, s_0 is the standard error of the adjustment and f the degree of freedom. If the above formulas are extended to take care of the weight P as well the formulation will be.

$$\underset{n\times 1}{L} + \underset{n\times 1}{V} = \underset{n\times m}{A}\ \underset{m\times 1}{x} \qquad (4.5)$$

$$x = (A^tPA)^{-1}A^tPL \qquad (4.6)$$

$$V = Ax - L \qquad (4.7)$$

$$Q_{vv} = Q_{ll} - A(A^tPA)^{-1}A^t \qquad (4.8)$$

$$s_0 = \sqrt{\frac{V^tPV}{f}} \qquad (4.9)$$

$$\sigma_{vi} = s_0\sqrt{Q_{vvii}} \qquad (4.10)$$

For further information, see Bjerhammar, 1973.

In the adjustments used in this thesis a gross error detection has been done with the data snooping method. In this method a test value is calculated and compared to a limit. If the test value passes the limit the observation is erroneous. A disadvantage of this method is that it can only detect one erroneous observation. The test value is calculated as follows.

$$w_i = \frac{\|v_i\|}{\sigma_{vi}} \qquad (4.11)$$

where v_i is the residual of the observation i and σ_{vi} is the standard error of the residual of the observation. For details on gross error detection and variance component estimation methods used, refer to Geotec GmbH, 1992 (the handbook for the program PANDA) and Pelzer, 1985.

5. Measurements in Santiago de Puriscal

In Santiago de Puriscal observations have been done in the same network during several epochs since 1990. The observations have been divided into horizontal and vertical networks. Therefore this chapter covers traverse observations first and then the levelling. For point locations in the village see Appendix 1.2.

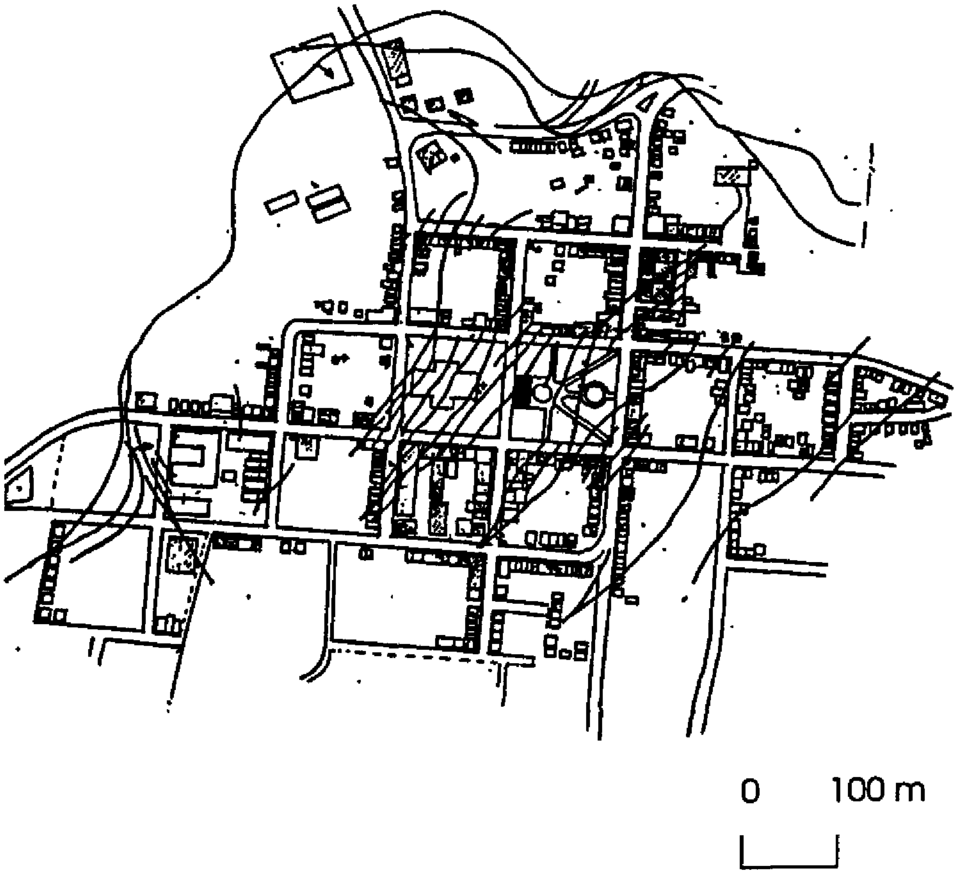


Figure 5.1. City plan of Santiago de Puriscal.

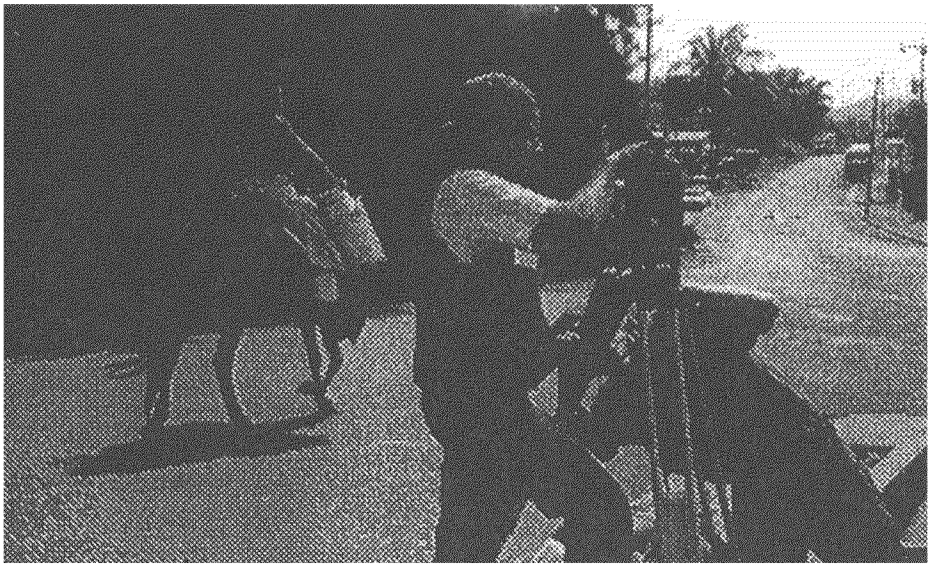
5.1. Instruments and traverse observations

The traverse observations have been done in four epochs. During the various epochs mostly the same points have been included in the network, but between May 1991 and August 1992 several points in the town disappeared or were destroyed. For this reason some auxiliary points were made in August 1992 to make observations possible.

Epoch	Measured by	Points included	Instruments used and their a priori error	Remarks
October 1990	Tomás Marino	All except 18, 29, 19 and 16	WILD T2000 (0.15 mgon) WILD DI3000(5mm+1ppm)	Not a network, but only traverses. Two parts without contact.
March 1991	Tomás Marino OVSICORI	All	WILD T2000 (0.15 mgon) WILD DI3000(5mm+1ppm)	Distances measured in one direction
May 1991	ETCG	Only 9, 3, 28, 1, 2, 20,21, 18, 15, 13, 16 and 17	ZEISS TH2 (0.2 mgon) ZEISS ETH (0.2 mgon) ELDI D4 (5mm+3ppm) Invar Base (>1cm)	Only points in the town centre.
August 1992	Tomás Marino Magnús Agnarsson Thomas Dubois	All except 29,28 and 7	WILD T2000 (0.15 mgon) WILD DI3000(5mm+1ppm)	Distances measured in both directions

Table 5.1. Traverse observations in Santiago de Puriscal.

For systematic errors in the instruments used, no corrections were done due to lack of information, except for atmospheric influence. Only the additive error of the DI3000 is known, see table 5.2 below. Errors like scale- and periodic error of the EDM:s are not known. As scale- and periodic errors change with time, it would have been valuable to have calibration values to correct the measured values with. In the case of the DI3000 the influence of those errors probably is not very big as the time passing is relatively short. This is a bigger problem considering the distances measured by ELDI D4, where no information on systematic errors is available.



Picture 5.1. Traverse observations in Santiago.

Time of calibration	result (mm)	standard error of calibration
December 1990	-0.34	0.58
April 1991	-1.71	0.43
November 1991	1.56	0.70
June 1992	-1.46	0.64

Table 5.2. Results of calibration of additive error for DI3000. It can be seen that the results are lower than the a priori error (3 mm +1 ppm) and thus may be neglected.

For the first three epochs no information is available about gross error control in the field, except that the difference between each circle should not pass a certain limit. This limit is not known to the authors. During the August 1992 epoch a continuos control was done. As soon as a traverse existed it was checked for any gross misclosures. If the misclosure passed 7 mm it was considered too big and the error was sought by the help of Brönnimans method. This method is used to find the co-ordinates of the point containing the error. The input is the known and the measured data, the result is the approximate co-ordinate of the erroneous point.

$$x = \frac{x_n + \underline{x}_n}{2} - \frac{y_n - \underline{y}_n}{2}\cot\left(\frac{\varphi_n - \underline{\varphi}_n}{2}\right)$$

$$y = \frac{y_n + \underline{y}_n}{2} - \frac{x_n - \underline{x}_n}{2}\cot\left(\frac{\varphi_n - \underline{\varphi}_n}{2}\right) \quad (5.1)$$

x, y, φ: calculated data

$\underline{x}_n, \underline{y}_n, \underline{\varphi}_n$: known data

x, y : the benchmark with error

Brönnimans method (Bjerhammar, 1967).

After finishing the observations all stations suspected to have gross errors were re-measured. The new observations were found to be satisfactory when they were controlled by the above mentioned method.

In the field the following quantities were measured; horizontal and vertical angles in two series, slope distances, instrumental and target height, pressure and temperature. The atmospheric correction was done with the following formula.

$$\Delta D_1 = 281.5 - \frac{0.29035 * p}{1 + 0.00366 * t} \quad (5.2)$$

In formula 5.2 the atmospheric correction for WILD DI300 is shown. Where p is pressure in mb and t is temperature in °C, ΔD_1 is in ppm (parts per million).

5.2. Adjustments of traverse observations

To be able to compare the observation data from the different epochs it is most common to calculate the co-ordinates of each point. In order to optimise the results, an adjustment of all observation data is done. The adjustment gives the co-ordinates of each point for each epoch. The theory of adjustment is described in chapter 4. To be able to calculate co-ordinates for the points it is necessary to establish a datum, i.e. an origo and an azimuth of the co-ordinate system. As in Puriscal no co-ordinates for the points are known, a local co-ordinate system was established where the origo was set to the point 1 and the X-axis through point 1 and 2, negative towards 2. This means that negative co-ordinates will exist in the network and that the X-axis does not coincide with the true north (geodetic north).

Actually the X-axis coincides fairly well with the true south. Thus in all figures of traverse adjustment the south is upwards.

To do a proper analysis of deformation it is necessary to do a free adjustment of the network. In the case of Santiago de Puriscal partly because no stable external points could be used and thus no points be used as datum defining without prior testing. If it is not possible to calculate the network as a free network, two points must be error free and thus bigger error ellipses are constrained in other parts of the network (see chapter 6.4). To be able to do this a program that allowed free network adjustment was needed. A copy of a German program called PANDA, which allows free network adjustments, was obtained to do the calculations. The program was kindly provided through the help of Prof. Wolfgang Niemeyer, Universität Braunschweig and Dipl. Ing. Dieter Tengen, GMBH GeoTec. PANDA is especially developed to do deformation analysis in 1, 2 and 3-dimensions. It can use horizontal- and vertical angles, azimuths, slope distances, horizontal distances, levelling data, measured co-ordinate differences and even GPS-data in the adjustment.

Prior to the adjustment all distances were corrected for atmospheric influence (see formula 5.2) and reduced to the horizontal plane. For the first adjustment the a priori standard errors, as given by producer of each instrument were used. For theodolites higher though, due to short distances. After the first adjustment gross errors were sought. The free parameters for the adjustments are translations in X and Y, one rotation and scale, thus giving a rank defect of 4 (see chapter 1.2).

5.2.1. The October 1990 network

When investigating the 1990 traverse observations, it was found that they were insufficient to use for any determinations. Only 11 angle- and 22 distance observations exist. Since they are spread out over the network, it is impossible to create any traverses as can be seen in figure 5.1

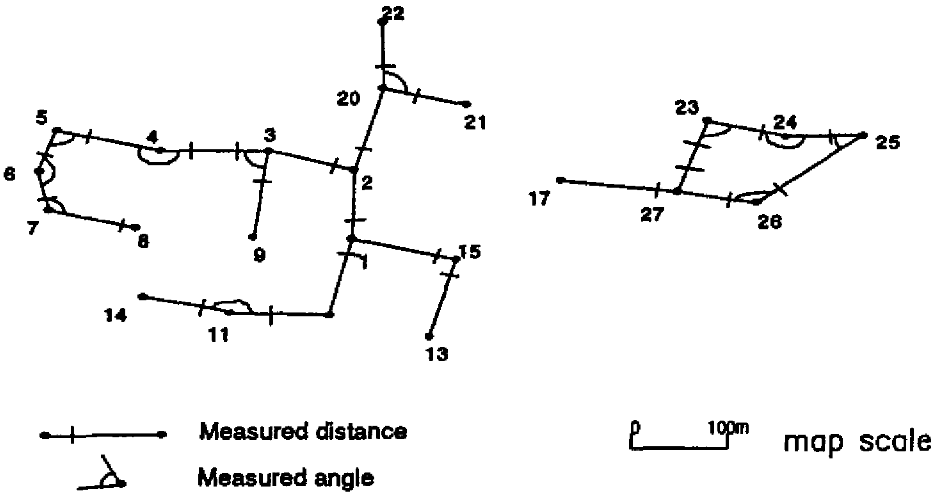


Figure 5.1. Design of the 1990 traverse observations.

All distances except for two are measured only once. This gives no room for any degrees of freedom, so not even a comparison between single observations can be done with any confidence. Due to the above mentioned reasons, the 1990 traverse observations have been excluded from the calculations.

5.2.2. Adjustment of the March 1991 network

In the network no gross errors, except several resulting from misunderstanding of raw data, could be found so the procedure could proceed. The a priori standard errors were changed according to the estimated variance components of the PANDA program. In the final calculation the a priori standard error for the T2000 theodolite was set to 1.0 mgon and the a priori standard error of the DI3000 EDM was set to 2.0 mm + 1 ppm.

The results of the adjustment were the following:

Number of observations	129
Number of points in network, all being datum defining points	27
Number of unknowns	54
Number of additional parameters	1
Number of unknown orientation elements	25
Rank defect	4
Degrees of freedom	37
A priori standard error of unit weight	1.000
A posteriori standard error of unit weight	0.869
Test value	1.325
F-distribution value for 95% confidence	1.510

Table 5.3. Results of adjustment March 1991.

The empirical test value of the network satisfies the theoretical value from the F-distribution. The configuration of the network and the error ellipses after adjustment can be seen in the following figure.

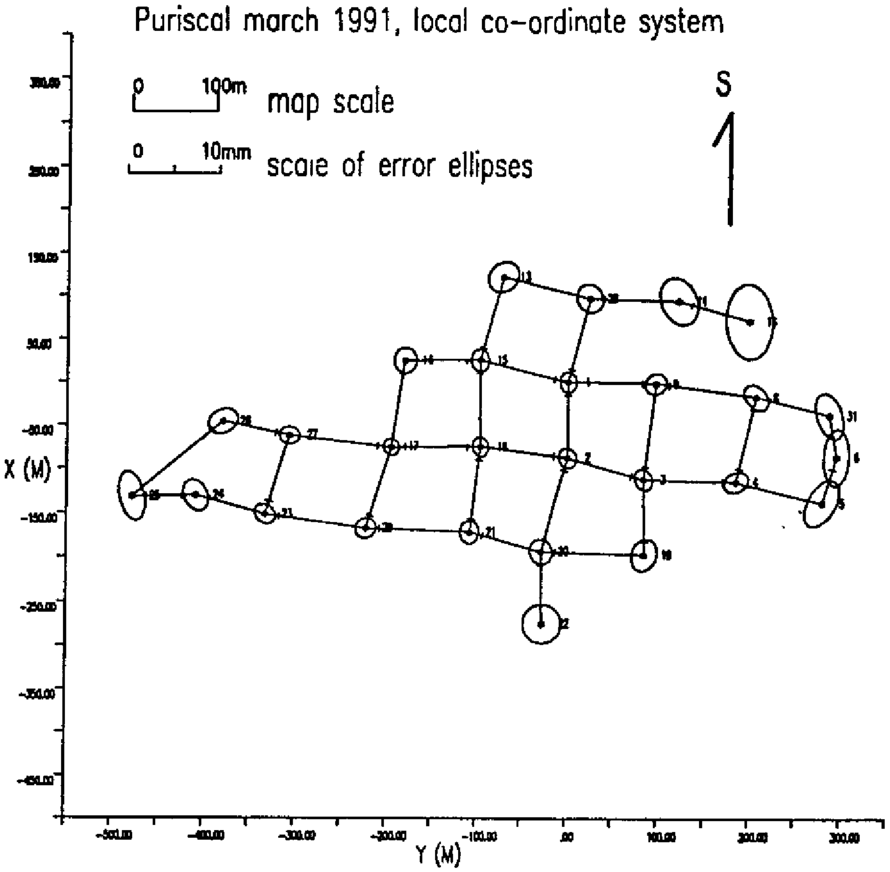


Figure 5.2. Results of March 1991 adjustment. Please note that south is upward in this figure.

5.2.3. Adjustment of the May 1991 network

For distances measured by invar bases the estimated error of the observations was used. In the network no gross errors were detected. Except a couple, due to misunderstanding of raw data, could be found so the procedure could proceed. Finally the a priori standard errors were changed according to the estimated variance components of the PANDA program. In the final calculation the a priori standard error for the TH2 theodolite was set to 1.0 mgon for the ETH3 to 1.2 mgon, for the ELDI4 12.0 mm and for the invar bases to 1.5 cm, except for one distance to 2.3 cm.

The results of the adjustment were the following:

Number of observations	57
Number of points in network, all being datum defining points	12
Number of unknowns	24
Number of additional parameters	2
Number of unknown orientation elements	12
Rank defect	4
Degrees of freedom	23
A priori standard error of unit weight	1.000
A posteriori standard error of unit weight	1.021
Test value	1.043
F-distribution value for 95% confidence	1.542

Table 5.4. Results of May 1991 adjustment.

The empirical test value of the network satisfies the theoretical value from the F-distribution. The configuration of the network and the error ellipses after adjustment can be seen in the following figure

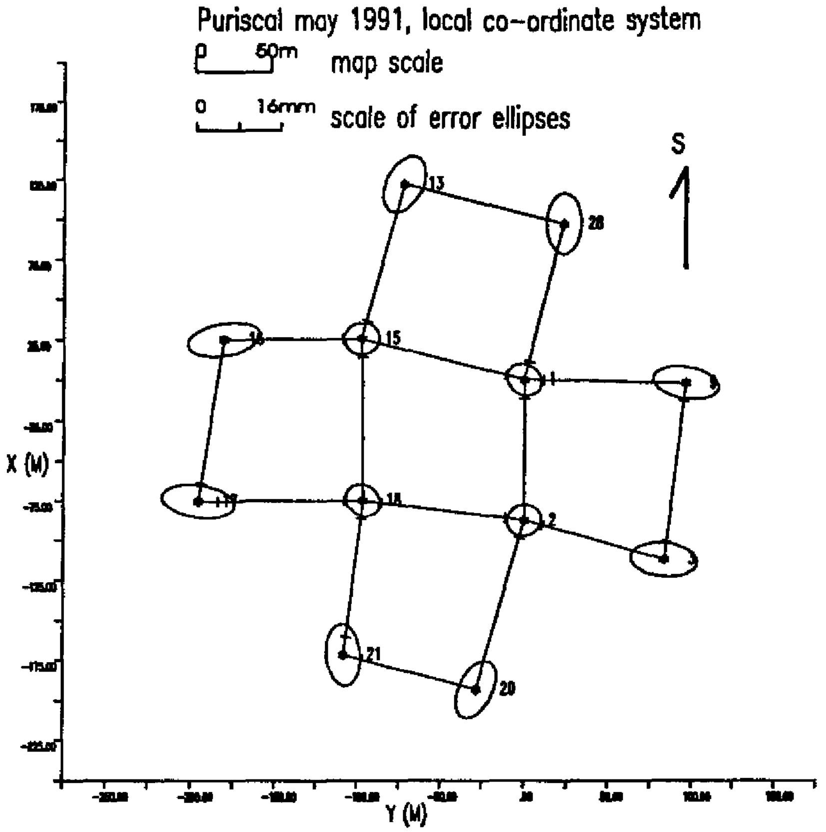


Figure 5.3. Adjustment of May 1991. Please note that south is upward in this figure.

5.2.4. Adjustment of the August 1992 network

In the network no gross errors, except some resulting from misunderstanding of raw data, could be found so the procedure could continue. The a priori standard errors were changed according to the estimated variance components of the PANDA program. In the final calculation the a priori standard error for the T2000 theodolite was set to 1.0 mgon and the a priori standard error of the DI3000 EDM was set to 2.0 mm + 1 ppm.

The results of the adjustment were the following:

Number of observations	106
Number of points in network, all being datum defining points	27
Number of unknowns	54
Number of additional parameters	1
Number of unknown orientation elements	25
Rank defect	4
Degrees of freedom	30
A priori standard error of unit weight	1.000
A posteriori standard error of unit weight	0.844
Test value	1.405
F-distribution value for 95% confidence	1.632

Table 5.5. Results of adjustment August 1992.

The empirical test value of the network satisfies the theoretical value from the F-distribution. The configuration of the network and the error ellipses after adjustment can be seen in the following figure

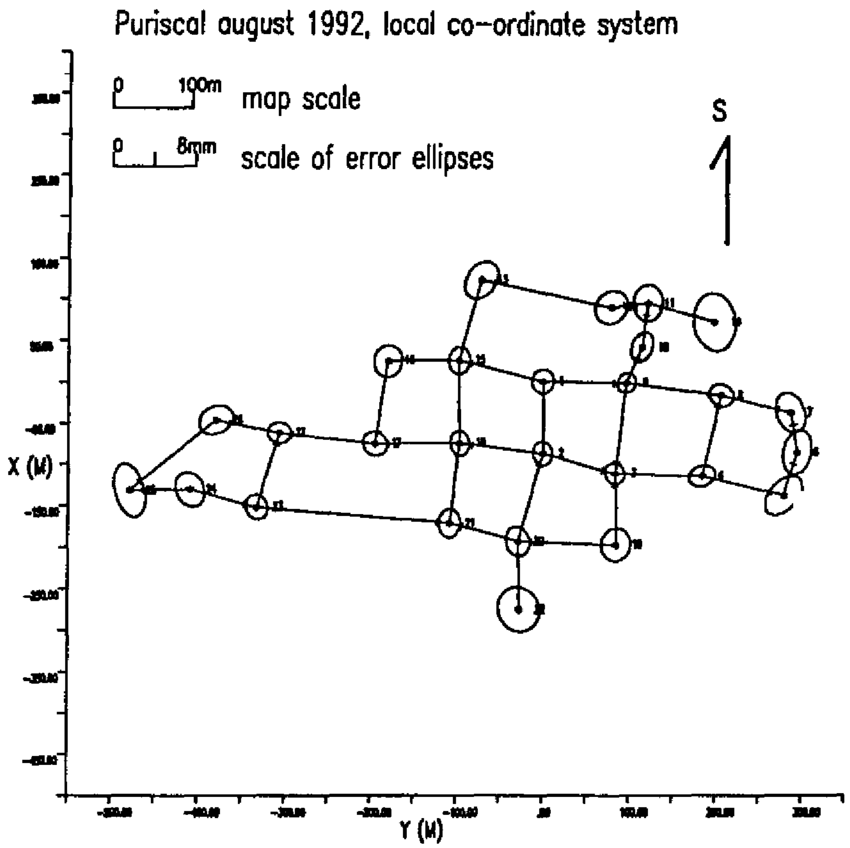


Figure 5.4. Adjustment of August 1992. Please note that south is upwards in this figure.

5.3. Precision levelling observations

Since maximum accuracy is needed, precision levelling was used to optimise the accuracy. The levelling was performed with the precision levelling instrument ZEISS Ni 2 and invar levelling rods. The observations have been done in almost full extension in three campaigns, April 1990, April-June 1991 and August 1992. The first two were performed by personnel from OVSICORI and ETCG, the last by the Authors. The points originates from a network made by Ing. Tomás Marino in April 1990.

Epoch	Measured by	Points included in calculations	Instrument used and its a priori error	Remarks
August-September 1990	G. Torres T. Marino L. Quesada	All	ZEISS Ni2 (0.3 mm/observation)	
March and June 1991	T. Marino	All except 19 and 31.	ZEISS Ni2 (0.3 mm/observation)	Connection of two epochs
July 1992	T. Dubois M. Agnarsson	All except 28, 29 and 31	ZEISS Ni2 (0.3 mm/observation)	Measured by the authors

Table 5.6. Level observations in Santiago de Puriscal.

The points are the same as used for the traverse observations. The same network was used for all epochs with a few exceptions, due to some points that were destroyed before the 1992 observation.

5.3.1. Control of instrument through collimation test in 1992

To make sure that the instrument functioned properly, a collimation test was made.

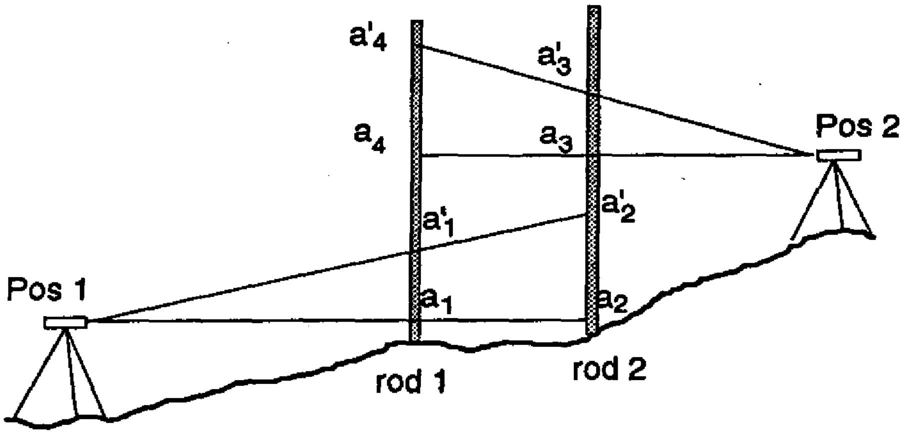


Figure 5.5. Collimation test.

The test consists of two observation sites and two levelling rod sites (see figure 5.5). The levels for levelling rod 1 and 2 are measured from position 1 and 2 separately. All four mentioned positions have to be in a line, and the distance in between should be a normal measuring distance, in our case around 20 m.

In figure 5.5, the obtained observations are made with an a'_i sign. The correspondent a_i indicates the level obtained with a perfect instrument. The test result is then calculated as follows:

$$a_4 = a'_1 - a'_2 + a'_3 \qquad (5.3)$$

$$\Delta = \frac{a'_4 - a_4}{2} \qquad (5.4)$$

Where the error $\varepsilon = \Delta$. For the used instrument, ε was determined to be 0.05 mm. That is low enough to be neglected, since the a priori accuracy of the used instrument is 0.3 mm.

5.3.2. Choice of measured level differences in 1992

The level difference between a point and its neighbouring points in every possible direction was measured. The aim was to create a network containing all earlier measured points in the village. In order to create an easily adjustable network, an effort was made to measure each point from at least two other sites. This was done with only two exceptions.

5.3.3. Measuring method in every separate levelling in 1992

To be able to control every single level difference, they were measured forward and back between the points. This made it possible to detect errors already in this early stage. A maximum misclosure between two points of 0.4 mm was tolerated. Since a levelling rod with a left and right scale was used, a check was also made in every single observation. In this case, the difference between right and left values were not to exceed 0.4 mm. The limits were chosen regarding the accuracy of the instrument (0.3 mm / observation).

5.3.4. Gross error detection and re-observations in 1992

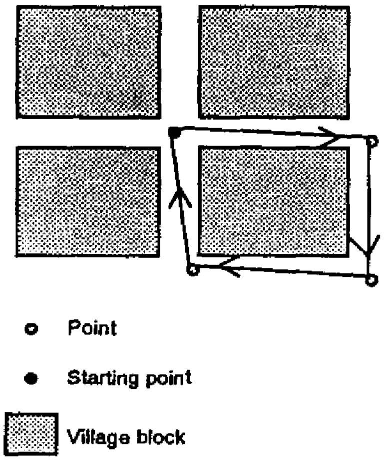


Figure 5.6. Small traverse structure.

While progressing observations, small traverses were made (mostly containing points in four street corners). The misclosure limit was set to 1.0 mm. If an error was detected, other traverses were made containing parts of the first one. This made it possible to detect the troublesome level difference, which then was re-measured. In the 1992 campaign, this occurred 5 times. Finally, a preliminary adjustment of all points was made. The condition adjustment without weights was used, and the standard error of the observations proved to be 0.19 mm. In comparison with the a priori accuracy (0.3 mm) of the instrument, the result is more than sufficient, and no more re-observations had to be done.

5.4. Adjustment of level observations

An adjustment is done, as in the case of traverse observations, to minimise the residuals and to get an approximation of their size. To be able to carry through the adjustment, it is necessary to obtain approximate levels for a network. For this situation, it means obtaining a level for at least one point.

Since no levels are known in this network, one has to decide an arbitrary start value for an arbitrary point. The start value does not have to correspond with the height above mean sea level, since absolute levels are of no interest in this investigation. Point number 1 was chosen and set to the level 1000 m. This level is not fixed, as the point is involved in the adjustment. Therefore, its resulting level differs a little from the start level. The calculations were carried through with the program PANDA, as described in chapter 5.2.

The given a priori accuracy of the instrument (for the Ni 2 set to 0.3 mm / observation) can not be used, since the desired unit is the a priori standard error (mm/km), and therefore dependent on number of observations, terrain type etc. Therefore, the a priori standard error had to be found by changing the default value (1.00 mm/km) and study its effect on adjustment- and deformation procedures.

The value of the standard error was altered to make the results suit the following deformation calculation. The reason for this was that the a posteriori standard errors of the epochs has to be similar to be accepted by the deformation analysis. The altering procedure was iterated until the deformation procedure accepted the input data. The free parameter is translation in Z, which sets the rank defect to one.

5.4.1. Adjustment of the September 1990 level network

Prior to the adjustment, averages were calculated for levels measured more than once. A preliminary adjustment was then done to detect gross errors.

errors. Apart from a few reading errors, no gross errors were found among the observations. The default value 1.00 mm/km for a priori standard error suited the following deformation calculations.

The finally used parameters and results were as follows:

A priori standard error (mm/km)	1.00
A posteriori standard error (mm/km)	1.27
A priori variance for weight unit	1.000
Test value and a posteriori variance for weight unit	1.603
Value for F-distribution	1.798
Degrees of freedom	11
Number of observations	37
Number of datum defining points (all)	27
Rank defect	1

Table 5.7. Results of September 1990 adjustment.

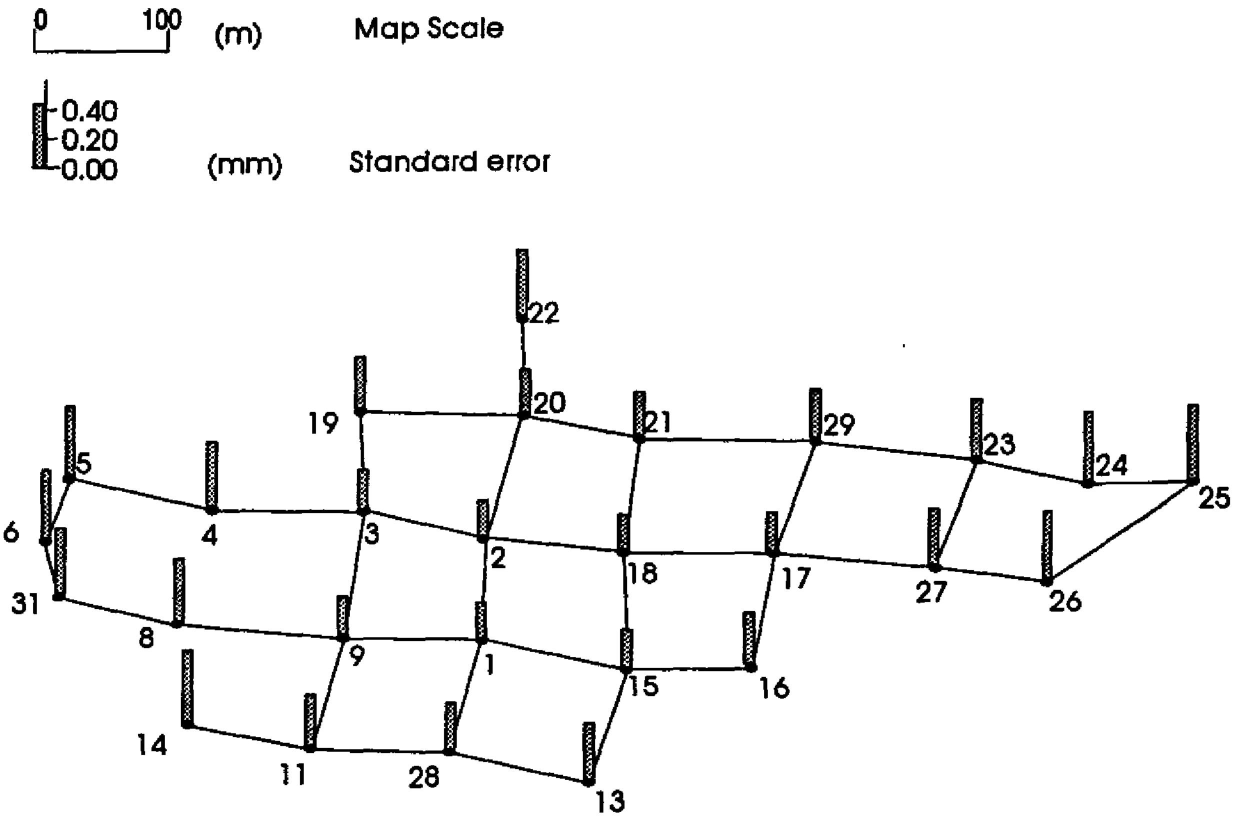


Figure 5.7. Standard errors for the 1990 network after adjustment.

5.4.2. Adjustment of the April and June 1991 level network

Before the adjustment, the observations were treated as described for the 1990 observation. 2.00 mm/km as a priori standard error was chosen (see chapter 5.3). For this epoch, some remarks have to be done:

- Point 31 was excluded from the calculations due to a misclosure of 0.2237 m for a small traverse containing points 4, 5, 6, 8 and 31. After comparisons with observations form the other epochs, the gross error was found to be in the observations containing point 31, which therefore was excluded. Point 19 was excluded for the same reason.

- The 1991 observations were done in two different campaigns; April and June. As a separate adjustment leaves lots of "loose ends" for the June network, the two campaigns have been connected. Separate adjustments would also generate worse conditions for deformation studies, because of the few points involved.

The used parameters and results were as follows:

A priori standard error (mm/km)	2.00
A posteriori standard error (mm/km)	2.65
A priori variance for weight unit	1.000
Test value and a posteriori variance for weight unit	1.755
Value for F-distribution	1.840
Degrees of freedom	10
Number of observations	34
Number of datum defining points (all)	25
Rank defect	1

Table 5.8. Results of April and June September 1991 adjustment.

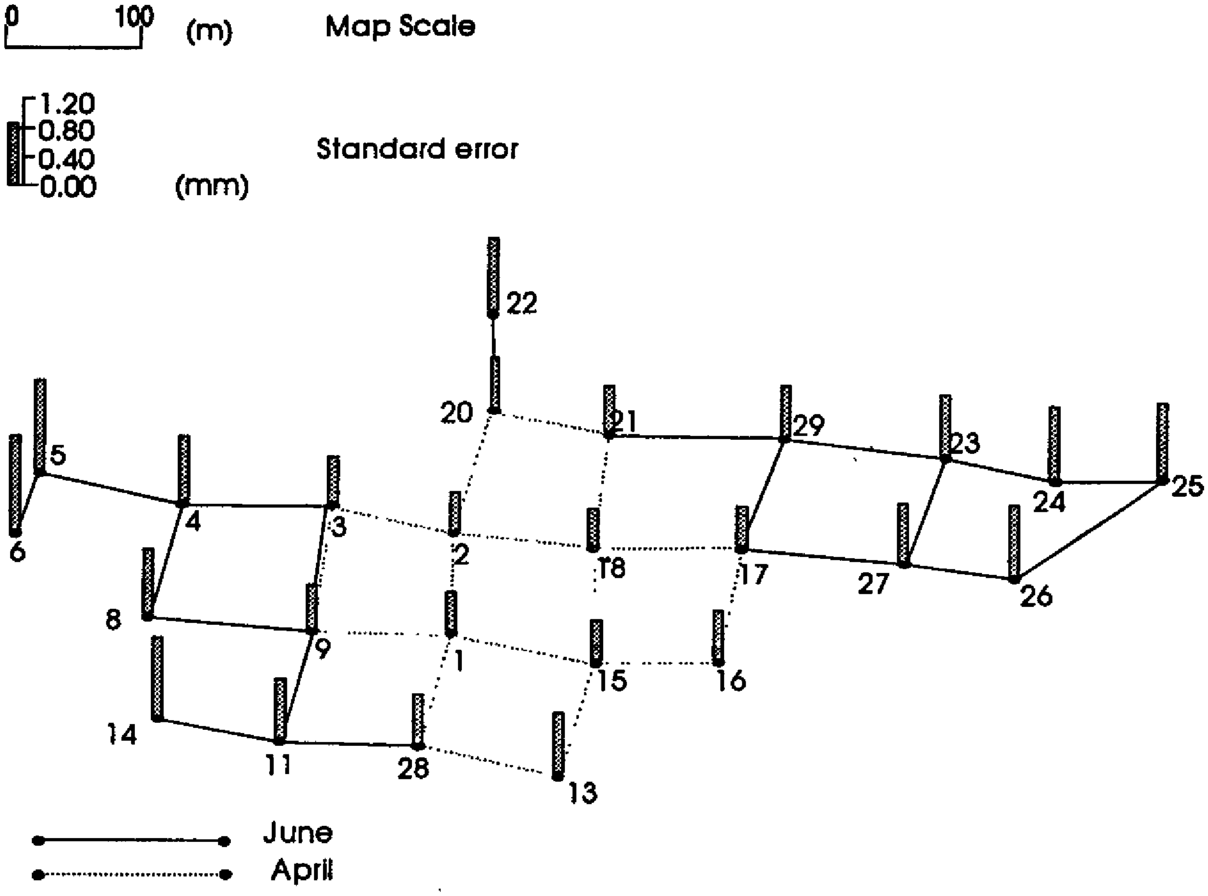


Figure 5.8. Standard errors for the 1991 network after adjustment.

5.4.3. Adjustment of the July 1992 level network

The same method as described for the earlier epochs was used. One remark is to be given:

- The destruction of points 28, 29, and 31 resulted in longer distances between some points. These observations were needed to keep the network together. However, no loss of accuracy was found in the results due to these circumstances. A priori standard error was set to 0.50 mm/km.

The used parameters and results were as follows:

A priori standard error (mm/km)	0.50
A posteriori standard error (mm/km)	0.62
A priori variance for weight unit	1.000
Test value and a posteriori variance for weight unit	1.533
Value for F-distribution	1.948
Degrees of freedom	8
Number of observations	31
Number of datum defining points (all)	24
Rank defect	1

Table 5.9. Results of July 1992 adjustment.

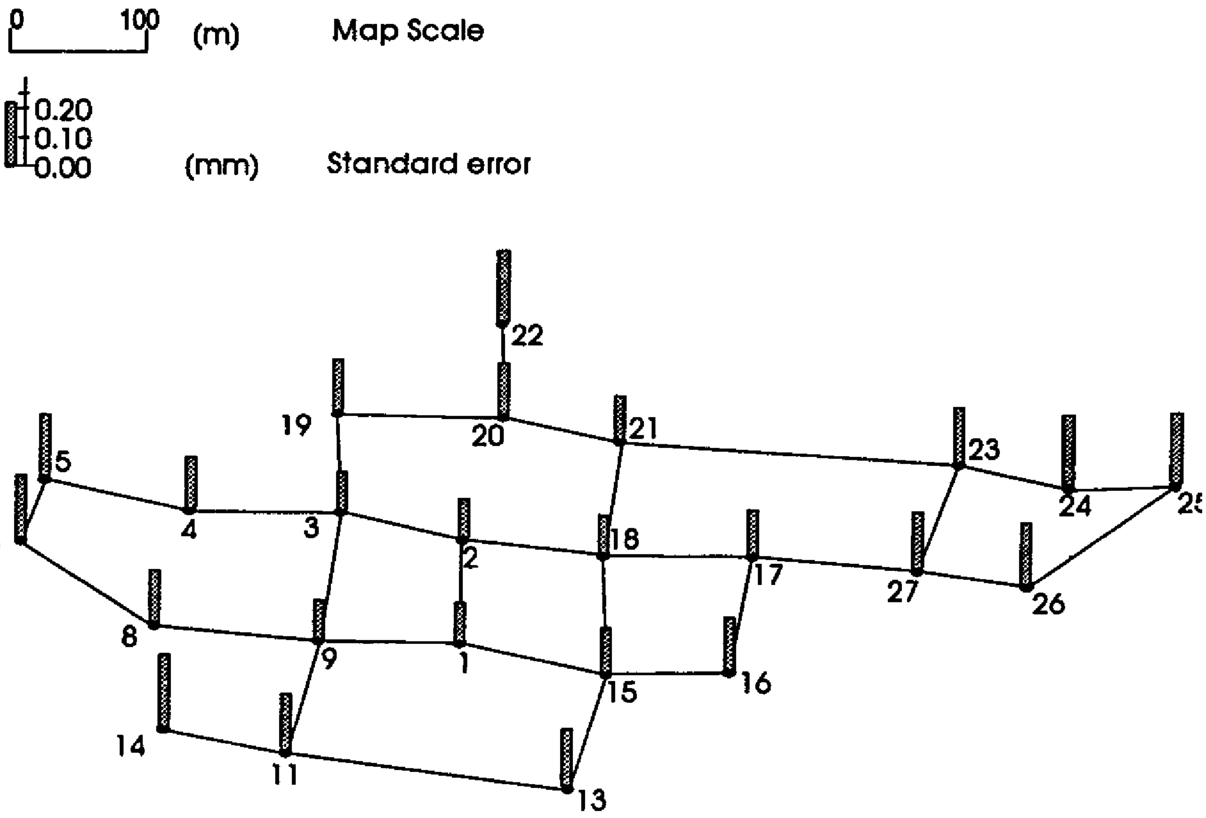


Figure 5.9. Standard errors for the 1992 network after adjustment.

5.4.4. Adjustment result interpretation

The adjustment results show that the results of the three epochs differ quite a lot in accuracy. The largest residuals are found in the 1991 epoch.

reason for this may be that the epoch is divided in two parts with two months in between. That interval may have been long enough to allow movements to affect the adjustment. The network has been held together though, since it gives the best analysis conditions (see chapter 5.4.2).

Generally the accuracy of the results are in the sub-millimetre level, which for this application provide good conditions for deformation detection.

The large residuals in each epoch are distributed to the boundaries of the network. This is normally the case due to fewer observations for points in the outer parts of the network.