

## SECTION 6

### DYNAMIC PARAMETERS

Table 6-I compares the periods and amplitudes of the ambient measurements resulting from an analysis using the methods described in Section 5 with those from finite-element dynamic analysis, and with code-formula estimates. The finite-element analysis is preliminary, reflecting engineering judgment as to the flat-slab width, effective shear wall stiffness, and effective coupling-beam moment of inertia.

The wind was very light during the measurements of Buildings A and B but was 35 knots during the measurements of Buildings C and D. This is evidenced by heavier modal participation in the lower frequencies for Buildings C and D. The concentration of spectral energy in the lower frequencies in Buildings C and D made the higher frequencies in Buildings C and D more difficult to identify.

Lengthy ambient vibration records improved power spectrum estimation by allowing a fine frequency resolution while averaging several spectra together. Most spectra were 2048 points long and were averaged up to 28 times. This enhanced the estimation of the natural periods, and damping ratios. The output files from program PEAK, which analyzed the power spectrum and phase spectrum data files, are shown with their corresponding spectra in Appendix B.

Figures 6-1 to 6-10 illustrate the "operational deflected shapes" (ODS) of the four buildings. Note that these are not normalized mode shapes since the spectral amplitude of ambient vibration measurements cannot be normalized by the spectral excitation amplitudes. "Operational deflected shapes" is the term used by the modal analysis community to describe the spatial description of the amplitude of vibration at resonant frequencies under operating conditions. Operational deflection shapes corresponded well to engineering judgment. That is,

for the most part, the number of inflection points in the ODS is one less than the number of the associated resonant frequency for that ODS. Furthermore, ODS amplitudes are larger in the lower resonant frequencies.

The root mean squared (RMS) displacement values presented in Table 6-I are not the first mode RMS values, but are computed using equation (5.20). RMS velocity and acceleration values, along with plots of acceleration response amplitude and phase spectra are given in Appendices B and C. Measured RMS accelerations were much less than the threshold of perception (0.005 g or 5 cm/sec<sup>2</sup> [Simiu and Scanlan 1986]).

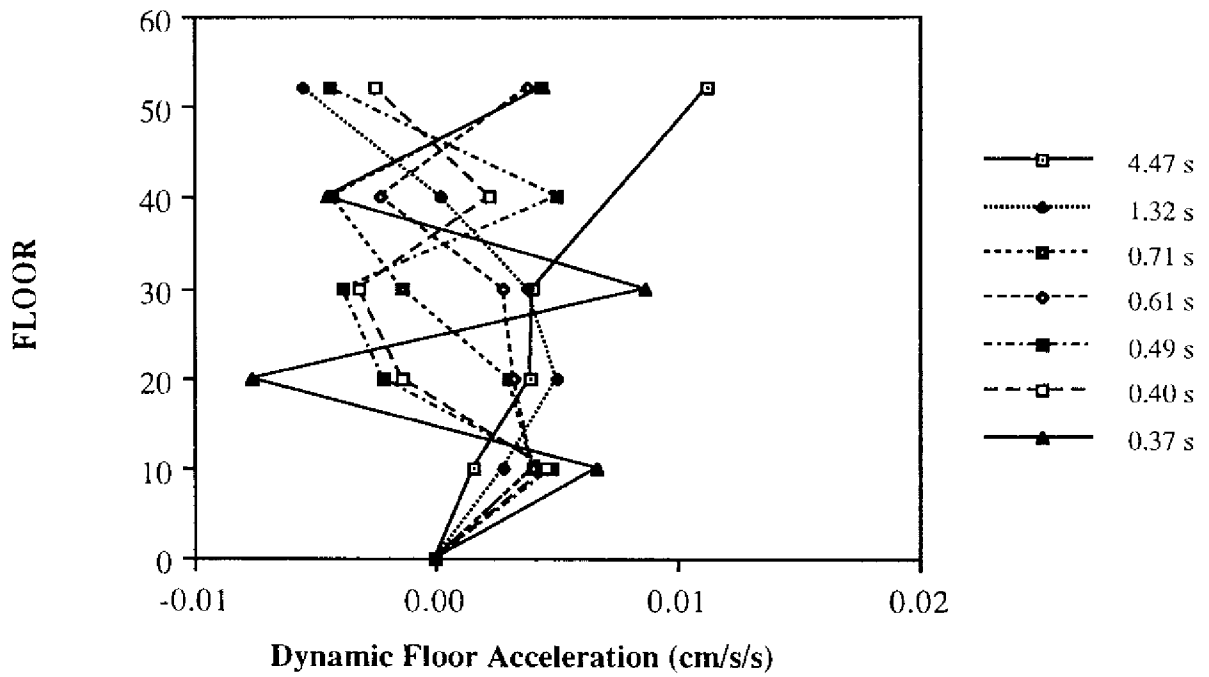
**TABLE 6-I Dynamic Parameters Estimated by Ambient Vibration  
Measurements, by Finite Element Analysis, and by Code.**

	<u>Measured Period (sec)</u>	<u>Measured Damping(%)</u>	<u>RMS Disp. (cm)</u>	<u>F.E.A.* Period (sec)</u>	<u>1988 U.B.C. Period(sec)</u>
<b><u>BUILDING A</u></b>					
1 N-S	4.47	2.0	0.0108	5.2	2.07
2 N-S	1.32	1.0		1.6	
3 N-S	0.71	0.2		0.8	
1 E-W	4.53	2.8	0.0071	4.6	2.07
2 E-W	1.23	1.0		1.3	
3 E-W	0.64	0.7		0.7	
1 TORSION	3.41	0.7		4.1	
2 TORSION	1.10	0.8		1.4	
3 TORSION	0.49	0.3		0.8	
<b><u>BUILDING B</u></b>					
1 N-S	3.98	2.4	0.0118	3.93	1.87
2 N-S	1.00	0.5		1.06	
3 N-S	0.57	0.8		0.59	
1 E-W	2.79	1.0	0.0066	2.63	1.87
2 E-W	0.80	0.7		0.80	
3 E-W	0.46	0.5		0.47	
1 TORSION	3.49	1.5		3.20	
2 TORSION	1.21	1.0		1.15	
3 TORSION	1.07	1.0		0.69	
<b><u>BUILDING C</u></b>					
1 N-S	1.94	1.4	0.0189	N/A**	1.25
2 N-S	0.66	0.7			
1 E-W	1.69	2.7	0.0063		1.25
2 E-W	0.57	1.2			
1 TORSION	1.32	1.3			
2 TORSION	0.70	0.9			
<b><u>BUILDING D</u></b>					
1 N-S	3.76	4.7	0.0520	N/A**	1.99
2 N-S	1.26	1.2			
3 N-S	0.68	0.6			
1 E-W	3.88	5.6	0.0825		1.99
2 E-W	1.17	1.1			
1 TORSION	2.91	2.4			
2 TORSION	0.83	0.7			

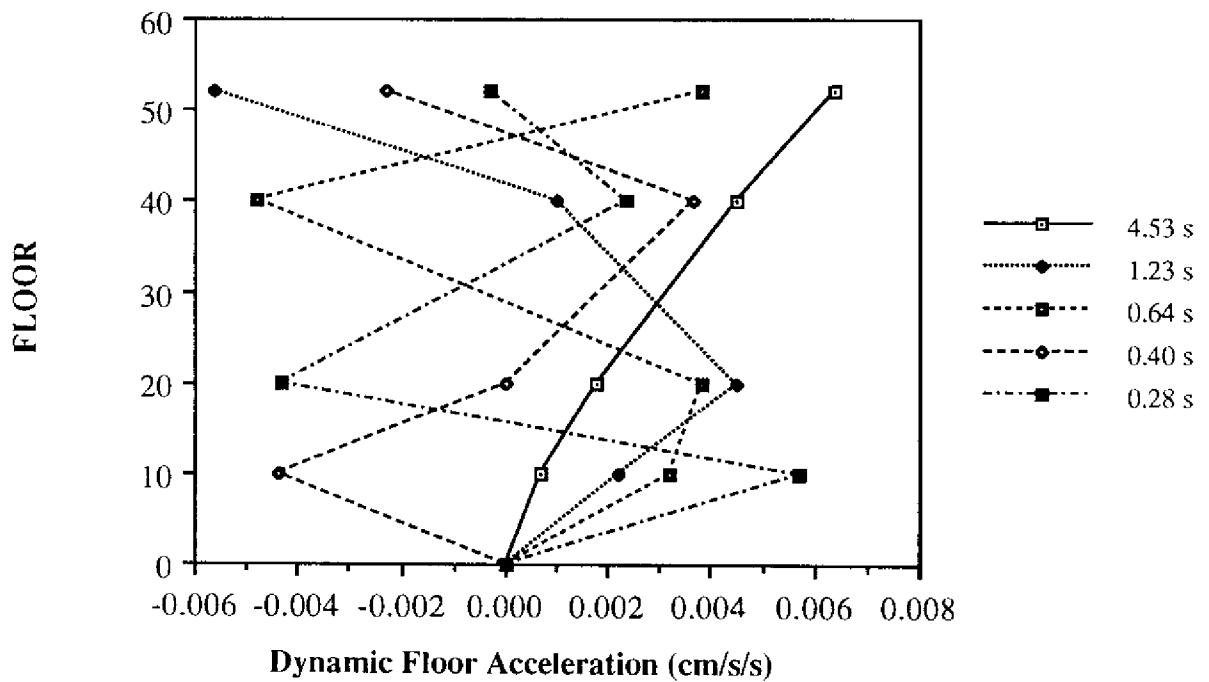
\* Computed periods were obtained using  $K_d=2.0$  in equation (7.1) for flat slabs and uncracked sections for columns, walls and beams [Grossman, in progress].

\*\* N/A indicates that these values are not available, this work is in progress by the designers.

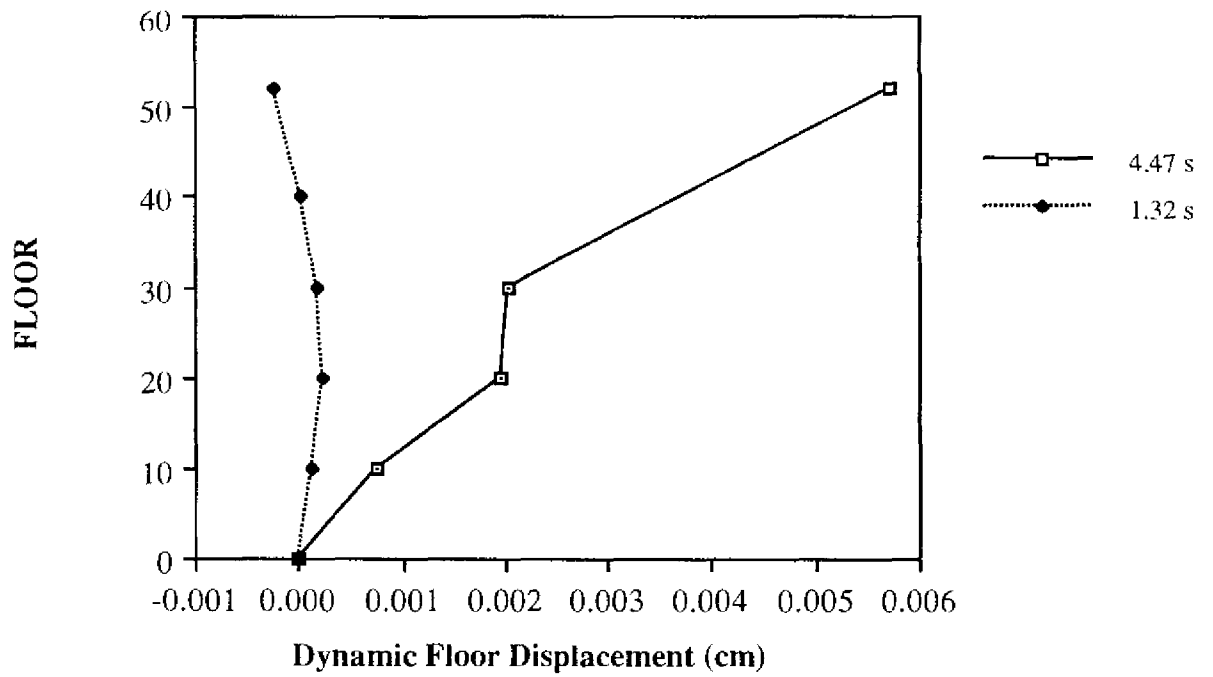
**Figure 6-1**  
**Building A; North-South Accelerations**



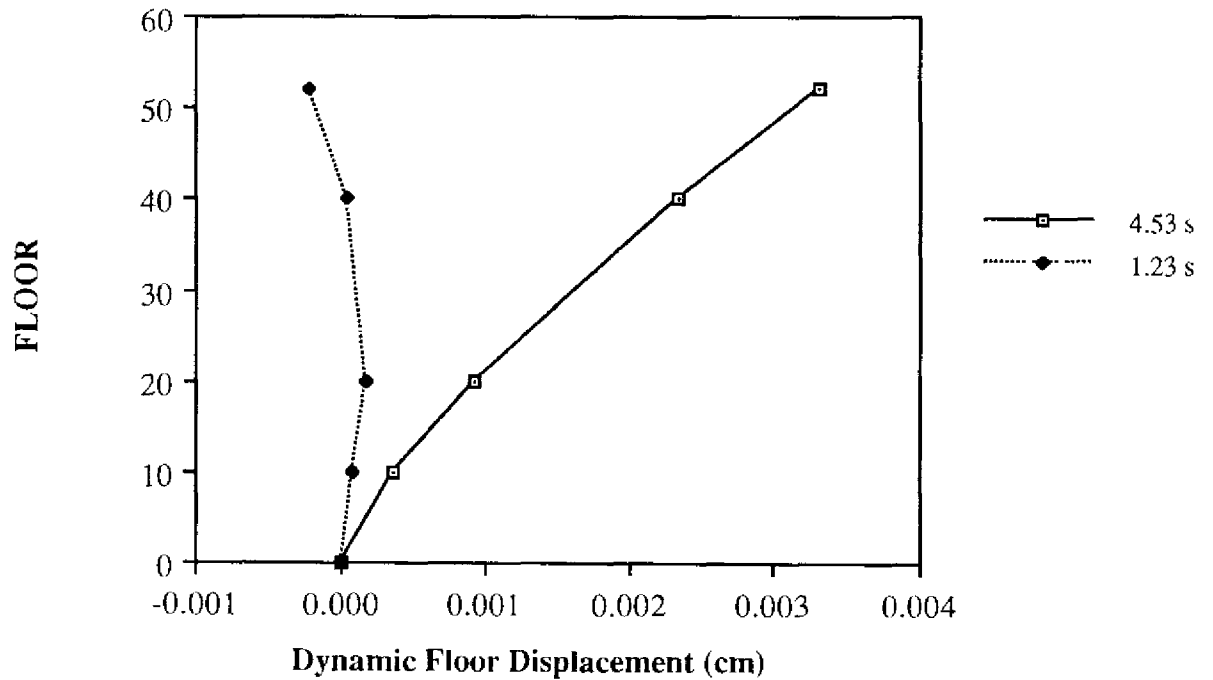
**Figure 6-2**  
**Building A; East-West Accelerations**



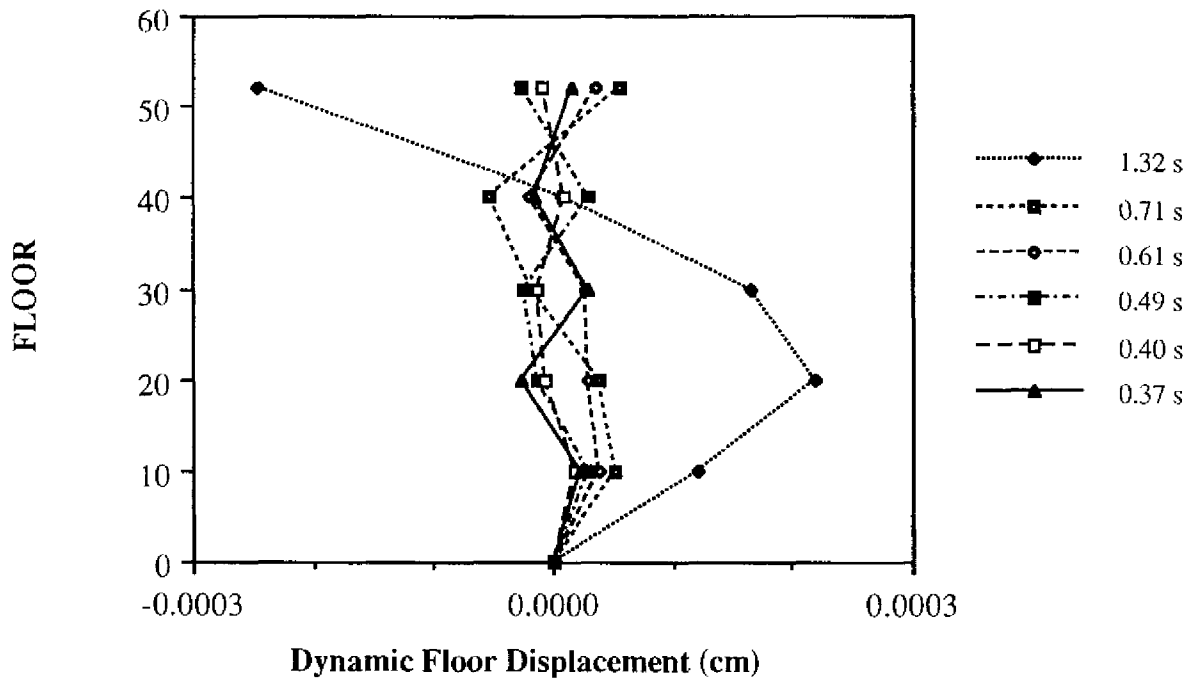
**Figure 6-3**  
**Building A; North-South Displacements**



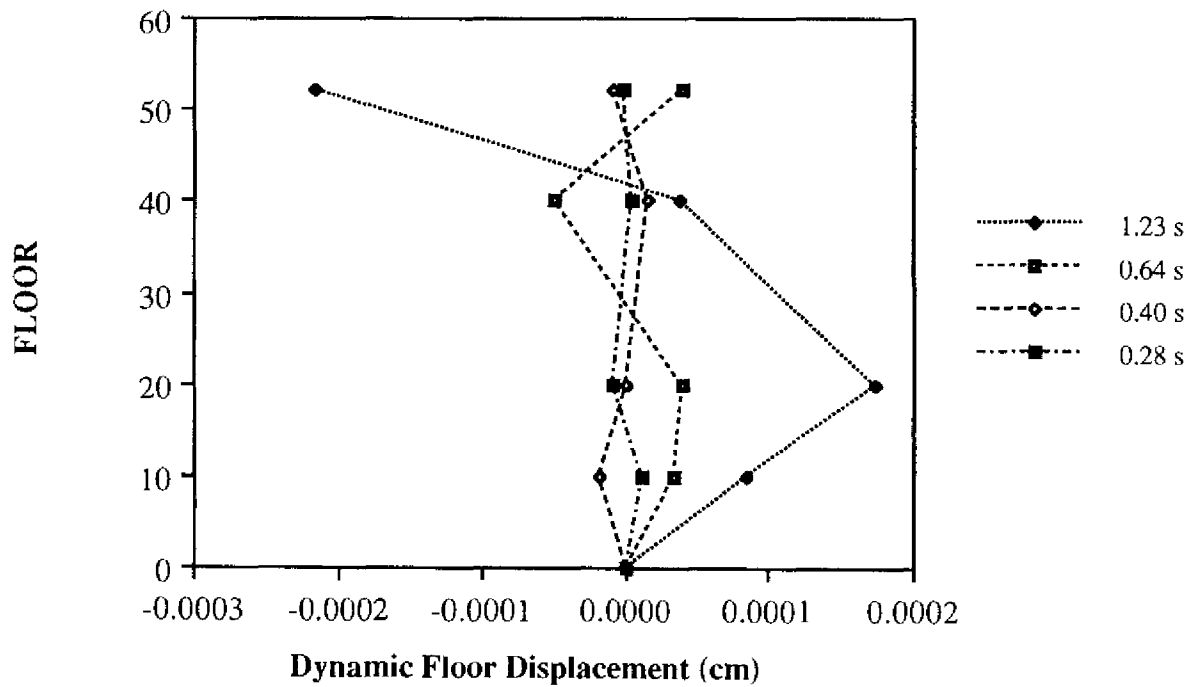
**Figure 6-4**  
**Building A; East-West Displacements**



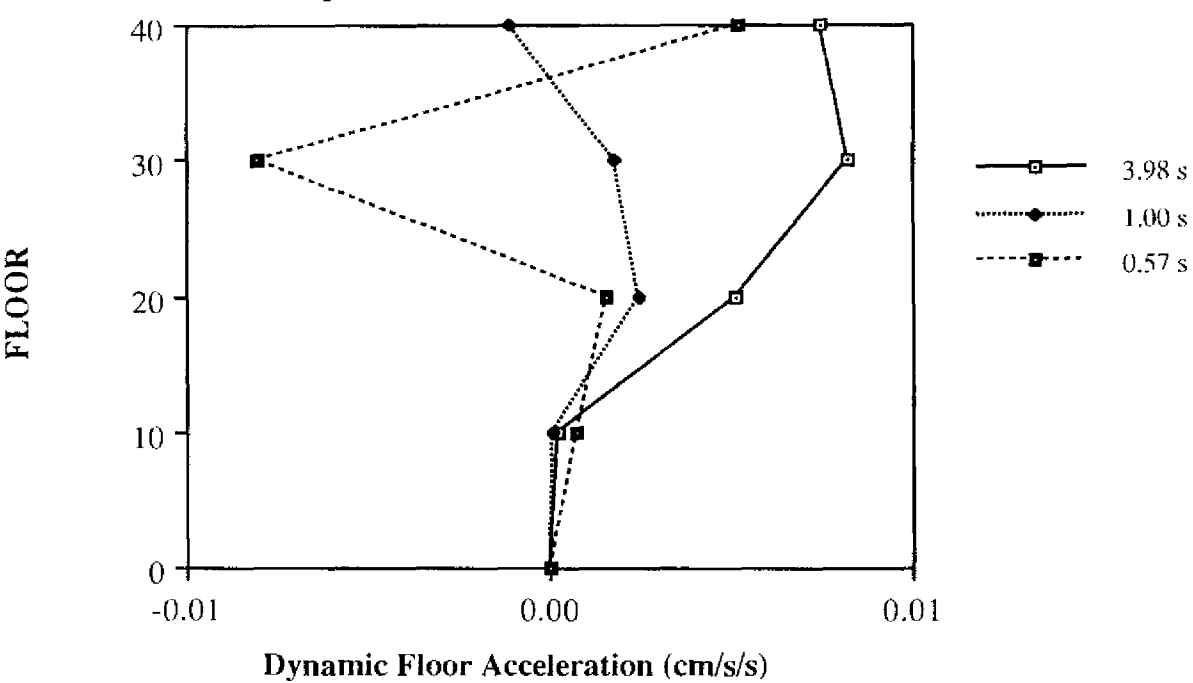
**Figure 6-5**  
**Building A; North-South Displacements**



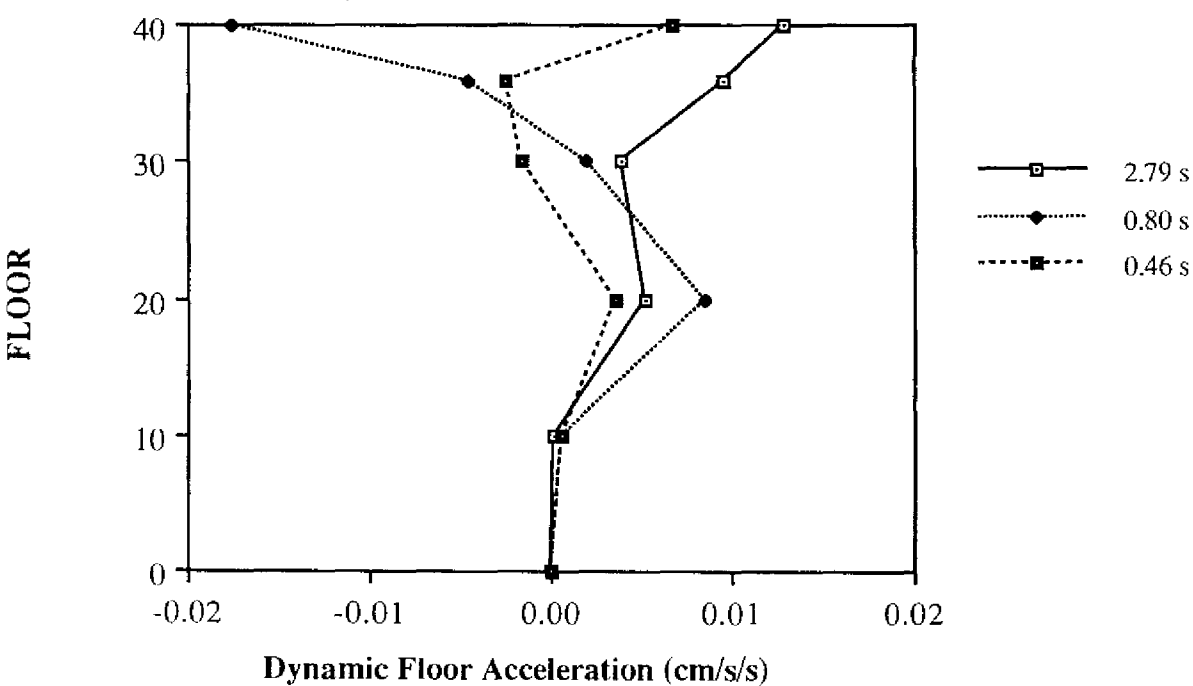
**Figure 6-6**  
**Building A; East-West Displacements**



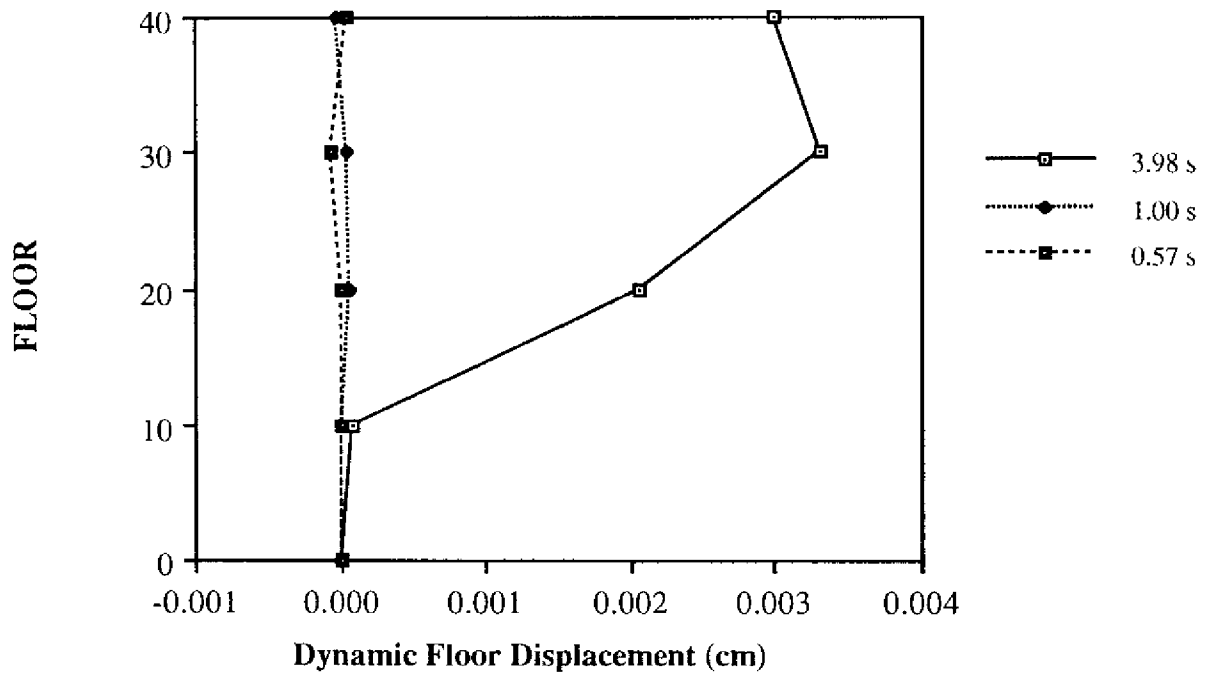
**Figure 6-7**  
**Building B; North South Accelerations**



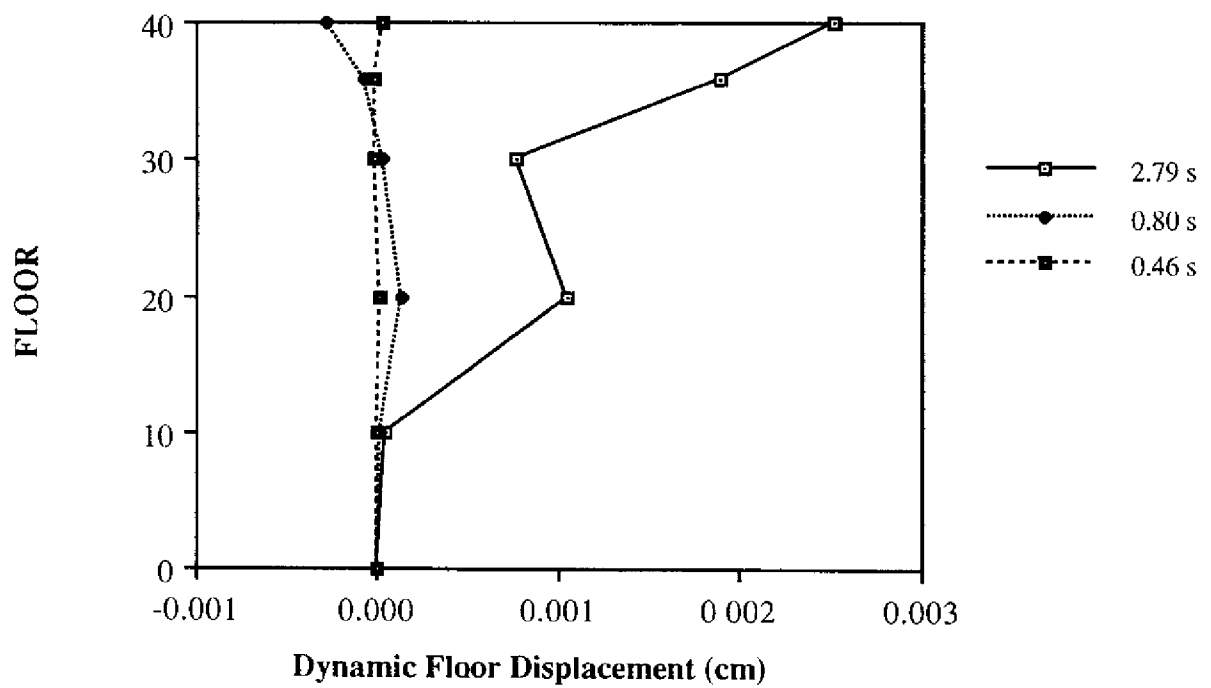
**Figure 6-8**  
**Building B; East-West Accelerations**



**Figure 6-9**  
**Building B; North-South Displacements**



**Figure 6-10**  
**Building B; East-West Displacements**





## SECTION 7

### RESISTANCE OF FLAT PLATE SYSTEMS TO LATERAL FORCES

In high-rise buildings that are constrained architecturally from the prevalent usage of windowless structural walls, buildings that have columns which do not fall in a regular grid pattern, and buildings that have a structural system determined by interior floor-plans, flat plates have been designed to resist lateral forces. Forming costs for flat plates are also less than those for beam-drops. However, the lateral resistance that flat plates provide is not clearly understood. The approach engineers should choose in the design of flat plate systems, when the flat plate is intended to provide lateral resistance, is not precisely specified in ACI 318-83 [ACI-318 Code 1983]. The Equivalent Frame Method was evaluated by the late Professor Vanderbilt [Vanderbilt 1981]. But disagreement in the 1983 ACI Code Committee 318 regarding its suitability resulted in the inclusion of both Equivalent Frame and Effective Width methods in the ACI 318-83 code. (See commentary [ACI-318 Commentary 1983] Section 13.3.1.2.) A lack of pertinent test data caused great uncertainty in the detailing of connections between plates and supporting columns in order for the plate to develop its full moment capacity. The discussions during the development of the 1983 Code concerning flat plate design for lateral forces led the Reinforced Concrete Research Council (RCRC) to assign researchers to tasks aimed at better understanding flat plate behavior. One of the tasks was motivated by the recent inclusion of moderate seismic requirements in many states East of the Rocky Mountains.

A methodology describing the effective width,  $\alpha l_2$ , of flat plates at interior supports [Grossman, 1987] was verified and adjusted [Grossman, in progress] to results of laboratory tests [Moehle 1990] is as follows (for notation consult ACI 318):

$$\alpha l_2 = K_d \left[ 0.3 l_1 + C_1 \frac{l_2}{l_1} + \frac{(C_2 - C_1)}{2} \right] \frac{d}{0.9 h} \quad (7.1)$$

with limits

$$0.2 K_d l_2 \leq \alpha l_2 \leq 0.5 K_d l_2 \quad (7.2)$$

where  $K_d$  depends on the load level and the computed drift index. Additional adjustments at the edge of the plate, and at side and corner supports were also recommended but not duplicated here. The  $K_d$  values were calibrated from the flat plate substructure tests [Moeble, 1990] and *tentatively* (pending additional field measurements) may be taken equal to:

At ambient conditions	$K_d = 1.5 - 2.0$
$H/800$	$K_d = 1.1$
$H/400$	$K_d = 1.0$
$H/200$	$K_d = 0.8$

A future goal of the current measurements and analyses of wind excited response of the four full-scale buildings will be to verify the appropriate values for  $K_d$  at smaller drift ratios ( $< H/800$ ) and to confirm the values of  $K_d$  interpolated from the substructure tests when possible. Computed dynamic properties (using  $K_d=2.0$ ) for two measured structures [Grossman, in progress] are summarized in Table 6-I.

More intimate knowledge of the fundamental periods and damping in actual flat plate structures is needed to better estimate seismic lateral loads, soil-structure interaction, and serviceability of tall structures. These issues will bear more weight when the need to retrofit existing structures (not designed for seismic loads), becomes pertinent. The research presented herein is the initial phase of work toward the task of correlating the design methods developed for flat-plate structures and shear walls with measured ambient dynamic properties. This correlation will implement ETABS models of the "as-built" condition of the measured models. Douglas and Ried [1982] calibrated similar SAP models to the measured bridge dynamic parameters, and their methods will be implemented here.

The approach starts by deriving a quadratic relationship between  $M$  parameters ( $P_i, i=1...M$ ) and  $N$  significant structural variables ( $X_k, k=1...N$ ). In this analysis, the parameters,  $P_i$ , are the natural periods. The structural variables,  $X_k$ , are the effective plate width, the effective shear wall area, and the effective coupling beam moment of inertia.  $M$  must be at least as large as  $N$ . Analytic parameters, obtained from an ETABS computation as a function of the structural variables, are denoted  $P_i$ ; while measured parameters, obtained from ambient dynamic measurements are denoted  $PM_i$ . The fitting of the ETABS model to the measured parameters,  $PM_i$ , by adjusting the structural variables,  $X_k$ , follows the following steps.

1. Specify a base-line value,  $X_k^b$ , an upper bound,  $X_k^u$ , and a lower bound,  $X_k^l$ , for each structural variable.  $X_k^l < X_k^b < X_k^u$ .
2. Run ETABS  $2N+1$  times. For each run, each  $X_k$  changes from  $X_k^b$  to  $X_k^l$  to  $X_k^u$  one at a time. Each run computes different set of  $M$  parameters (periods). Parameters computed by varying the structural variables are denoted  $PQ^i$ . There are three sets of parameters corresponding to each structural variable, corresponding to  $X_k^b$ ,  $X_k^l$ , and  $X_k^u$  respectively.

$$\begin{aligned}
 PQ_1^i &= P_i(X_1^b \dots X_N^b) \\
 PQ_2^i &= P_i(X_1^l \dots X_N^b) \\
 PQ_3^i &= P_i(X_1^u \dots X_N^b) \\
 &\vdots \\
 PQ_{2N+1}^i &= P_i(X_1^b \dots X_N^u)
 \end{aligned} \tag{7.3}$$

3. Derive a quadratic equation describing how each parameter,  $PQ^i$ , varies with changes in the  $N$  structural variables. The quadratic form of  $PQ^i$  is given by

$$PQ^i = C^i + \sum_{k=1}^N (X_k A_k^i + X_k^2 B_k^i) \tag{7.4}$$

The constant polynomial coefficients,  $A_k^i$ ,  $B_k^i$ , and  $C^i$ , of the quadratic can be found by solving

$$\begin{pmatrix} PQ_1^i \\ PQ_2^i \\ PQ_3^i \\ \vdots \\ PQ_{2N+1}^i \end{pmatrix} = \begin{bmatrix} 1 & X_1^b \dots X_N^b & X_1^{b^2} \dots X_N^{b^2} \\ 1 & X_1^1 \dots X_N^b & X_1^{1^2} \dots X_N^{b^2} \\ 1 & X_1^u \dots X_N^b & X_1^{u^2} \dots X_N^{b^2} \\ \vdots & \vdots & \vdots \\ 1 & X_1^b \dots X_N^u & X_1^{b^2} \dots X_N^{u^2} \end{bmatrix} \begin{pmatrix} C^i \\ A_1^i \\ \vdots \\ A_N^i \\ B_1^i \\ \vdots \\ B_N^i \end{pmatrix} \quad (7.5)$$

using, for example, singular value decomposition.

4. If each  $PQ^i$  can be put in terms of a quadratic function of the structural variables, the structural variables corresponding to the measured parameters can be found in a least squares sense. Using the polynomial expressions for  $PQ^i$  given by equations (7.4) and (7.5), an error function can be written

$$E^2 = \sum_{i=1}^M (PM_i - PQ^i)^2 \quad (7.6)$$

Equation (7.6) is minimized by setting the partial derivatives of  $E^2$  with respect to  $X_k$  equal to zero and obtaining  $N$  equations to solve for the  $N$  different structural variables,  $X_k$ . There are a variety of numerical minimization procedures which can accomplish this task.

5. Once a set of the structural variables has been identified, the procedure can be repeatedly refined by using the identified values of  $X_k$  as  $X_k^b$  and diminishing the difference between  $X_k^u$  and  $X_k^1$ . This will refine and reduce the uncertainty in the true structural parameters,  $X_k$ .

By correlating ETABS models to natural periods measured under various wind conditions, including very strong winds, and by including recent static tests on flat-plate/column models, it is anticipated that criteria for degradation of ambient stiffness to stiffness at load levels anticipated during seismic events will be established. Top story displacements during strong winds may be on the order of  $H/800$  or  $H/1000$ . Recent static tests on scaled flat plate

substructures indicates the degradation of lateral resistance for drifts corresponding to  $H/800$ ,  $H/400$ ,  $H/200$  and larger. The laboratory study fine tuned an empirical formula, equation (7.1), relating the effective width at interior supports of the flat plate to the load level, causing a defined top story drift. The purpose of the research at hand is to confirm the laboratory results at lower load levels by identifying the effective plate width of full scale structures subjected to various load levels. Once verified, this stiffness degradation can be used in seismic vulnerability assessment of many flat-plate and shear-wall structures East of the Rocky Mountains.

## SECTION 8

### CONCLUSIONS

1. Vibration measurement analysis of high-rise structures in ambient conditions is useful in revealing the differences between the as-built characteristics of the structure and the design representation of the building. The analyses of measurements from four reinforced concrete flat plate structures in Manhattan ranging from 27 stories to 52 stories are presented in this report. The aspect ratios of the structures ranged from 2.1 to 5.3. The four structures were in various phases of construction at the time of the measurements.
2. Using modular, synchronized (to within 1 millisecond), digital data recorders simplifies the instrumentation of large scale structures. These recorders do away with the task of running cables between remote sensors and a central recording unit. Hence, measurements can be taken unobtrusively and with no interruption in construction. All the structures were measured with triaxial force-balance accelerometers in at least four locations. Data was collected at 25 or 50 samples per second.
3. Ambient vibration measurements were analyzed using software tailored for this purpose. The auto-power spectrum and phase spectrum are estimated using fast Fourier transforms and a windowing method. Interpretation of the spectral estimates was enhanced by peak-picking software, also tailored for ambient structural vibration analysis.
4. Recording long lengths of ambient vibration data allows for several spectral averages which enhances spectral resolution. Curve-fitting the spectral peaks with a quadratic further improves the precision of the estimated periods. Measured ambient fundamental periods from all four buildings are shorter than those given by the 1982 Uniform Building Code, but are longer than fundamental periods estimated using formulas in the 1988 Uniform Building Code, and the proposed NYC Seismic Code. Fundamental

periods range from 1.69 seconds in the shortest building to 4.53 seconds in the tallest building.

5. Ambient vibration analysis using spectral analysis methods is robust. However results derived from spectral band-width measures, such as damping ratios, should be viewed with extreme caution. Bias and leakage errors in the spectrum estimates affects band-width measures strongly. Damping estimates in ambient conditions reported in this study range from 0.2% in the higher modes to 5.6% in the fundamental mode.
6. Three methods for assessing the artificial widening of spectral peaks due to averaging and windowing finite lengths of discrete data are reviewed. These methods can be used as explicit guidelines in choosing target frequency intervals, and required data record lengths for a specific application. Comparing damping ratios computed from the spectra of simulated acceleration records with the known damping ratios is the most robust method for choosing the proper level of frequency resolution and spectral averages. Future analyses using multi-taper spectral estimation methods, in which the data is repeatedly windowed with orthogonal weighting functions, should result in spectra with less bias.
7. The computation of root mean square (RMS) acceleration and RMS displacement from normalized power spectra of acceleration data was confirmed using a small-scale laboratory test in which acceleration and displacement were measured and recorded simultaneously. Root mean square displacements range from 0.007 cm under very light wind to 0.08 cm under stronger winds. The tallest buildings were measured only under light winds.
8. The first mode alone cannot account for the total motion of tall structures in ambient conditions. Indeed, the higher modes often participate more strongly in the lower floors. Dynamic measurements are very useful in evaluating the higher mode response of the structure. The participation of lower modes is heavier during strong winds. So

the total displacement response to strong wind can be estimated using the first mode alone within acceptable error margins.

- 9 Differences in periods estimated from ambient vibration measurements and preliminary finite element analysis can be reconciled by fitting uncertain quantities, such as effective flat-plate widths, in the finite element model. In so doing, the effective flat plate width can be related to lateral load level under service load conditions.
10. Fitting flat plate widths in the finite element model to measured periods can give a better understanding of the contribution of flat plates to the lateral resistance in tall buildings, and how this resistance changes with higher load levels.
11. The data presented herein is part of an on-going study of the behavior of flat-plate reinforced concrete structures. Fundamental periods from these tests will be compared to periods from future tests under different wind conditions or different stages of construction. Combining the results of these studies with the results of destructive tests of flat-plate substructures will lead to a method for evaluating the behavior of high-rise flat-plate structures at large drift ratios ( $H/200$ ).



## SECTION 9

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