APPENDIX A

DERIVATION OF TWO DAMPING ESTIMATORS

Textbook formulae for damping estimation using impulse response functions or spectral band-widths should be used with caution when applied to experimentally obtained discrete data. This appendix contains the derivation of these two common damping estimators and assesses their suitability to experimental data from large-scale structures [Beards 1983, Paz 1985].

A.1 Time Domain

In the time domain, damping is often estimated using the logarithmic decrement. Consider the SDOF displacement response to an impulse.

$$x(t) = A_0 e^{-\omega_n \zeta} \sin\left(\omega_n \sqrt{1-\zeta^2}\, t - \varphi\right) \tag{A.1}$$

This is a sinusoidal oscillation with exponentially decreasing amplitude. The response displacement reaches adjacent relative maxima at times t_1 and t_2, with displacements A_1 and A_2. $t_2 - t_1 = \tau$. The variable τ is the damped natural period.

$$\frac{A_1}{A_2} = \frac{e^{-\omega_n \zeta t_1}}{e^{-\omega_n \zeta (t_1+\tau)}} = e^{-\omega_n \tau} \tag{A.2}$$

By definition,

$$\omega_n \sqrt{1-\zeta^2}\, \tau = 2\pi \tag{A.3}$$

Defining the logarithmic decrement, δ,

$$\delta = \ln\left(\frac{A_1}{A_2}\right) \tag{A.4}$$

taking the natural log of both sides of equation (A.2), and substituting equation (A.3) results in an expression for damping.

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \tag{A.5}$$

If the damping is purely viscous then several oscillations may be used to compute the logarithmic decrement.

$$\delta = \frac{1}{n} \ln\left(\frac{A_1}{A_{n+1}}\right) \tag{A.6}$$

Equation (A.5) is exact. In its derivation, no expressions were truncated and no terms were ignored. If the damping is viscous, then δ will not depend upon the strain, deflection, temperature, or strain rate. In this case, δ will be constant with respect to the section of the data record chosen for analysis. In addition, the logarithmic decrement may be computed using acceleration amplitudes directly if the response is purely single mode. However, several damping mechanisms usually contribute to the total damping and each mechanism's contribution usually is *not* independent of strain, deflection, temperature, and strain rate. Often, damping ratios computed with the initial, large motion, response are larger than those computed using the subsequent, smaller amplitude, motion. Further experimental complications arise from the fact that impulse decay measurements are often polluted with higher mode response, and that low frequency noise can seriously distort the results. Additionally, large, purely impulsive forces are difficult to generate in high-rise structures and damping ratios for higher modes are very difficult to obtain using this technique.

A.2 Frequency Domain

Frequency domain damping evaluation, although simpler experimentally, requires many more mathematical assumptions than does evaluation of damping in the time domain.

Consider the dynamic amplification of a SDOF system responding to steady state harmonic excitation of circular frequency ω. If the response quantity is acceleration, the dynamic amplification factor is

$$\frac{\ddot{x}}{x_{st}} = \frac{\omega^2}{\sqrt{\left(1 - \Omega^2\right)^2 + \left(2\zeta\Omega\right)^2}} \tag{A.7}$$

where $\Omega = \omega/\omega_n$. The peak of the function described by equation (A.7) occurs when

$$\Omega = \sqrt{1 - 2\zeta^2} \tag{A.9}$$

and has the value of

$$a = \frac{\ddot{x}}{x_{st}} = \frac{\omega^2}{2\zeta\sqrt{1 - \zeta^2}} \tag{A.10}$$

The value of the acceleration frequency response function, equation (A.7), at the frequency $\Omega = 1 + \Delta\Omega/2$ is

$$b = \frac{\left(1 + \frac{\Delta\Omega}{2}\right)^2 \omega_n^2}{\sqrt{\left[1 - \left(1 + \frac{\Delta\Omega}{2}\right)^2\right]^2 + \left[2\zeta\left(1 + \frac{\Delta\Omega}{2}\right)^2\right]^2}} \tag{A.11}$$

Values for a^2 and b^2 are indicated graphically in Figure 5-1. Since the eventual aim is to compute damping estimates from power spectra, the ratio of the squares of equations (A.10) and (A.11) is

$$\frac{a^2}{b^2} = \frac{\left(1 - 2\zeta^2\right)\left\{\left[1 - \left(1 + \frac{\Delta\Omega}{2}\right)^2\right]^2 + \left[2\zeta\left(1 + \frac{\Delta\Omega}{2}\right)^2\right]\right\}}{4\zeta^2\left(1 - \zeta^2\right)\left(1 + \frac{\Delta\Omega}{2}\right)^4} \tag{A.12}$$

Solving for ζ^2, we obtain

$$\zeta^2 = \frac{(1 - \varepsilon_2)\left(\Delta\Omega + \frac{\Delta\Omega^2}{4}\right)^2 + \frac{a^2}{b^2}\varepsilon_1\varepsilon_4}{4\left(\frac{a^2}{b^2}\varepsilon_4 + \varepsilon_3 - 1\right)} \tag{A.13}$$

where

$$\varepsilon_1 = 4\zeta^4 \tag{A.14}$$

$$\varepsilon_2 = 4\left(\zeta^2 - \zeta^4\right) \tag{A.15}$$

$$\varepsilon_3 = \Delta\Omega + \frac{\Delta\Omega^2}{4} \tag{A.16}$$

and

$$\varepsilon_4 = \begin{cases} \left(1 + \frac{\Delta\Omega}{2}\right)^4 \text{ if the data is acceleration} \\ 1 \text{ if the data is displacement} \end{cases} \tag{A.17}$$

For small values of damping, $\varepsilon_1 < \varepsilon_2 < \varepsilon_3 < \varepsilon_4$. Neglecting higher order terms, such as $\Delta\Omega^2$ with respect to $\Delta\Omega$, and both $\Delta\Omega^2$ and $\Delta\Omega$ compared to 1, and assuming the power spectrum data is from an acceleration record, ε_1 , ε_2 , and ε_3 are set to zero, and

$$\zeta = \frac{\Delta\Omega}{2\sqrt{\frac{a^2}{b^2}\left(1 + \frac{\Delta\Omega}{2}\right)^4 - 1}} \tag{A.18}$$

which is equivalent to equation (5.11).

The truncated formula, (5.11) or (A.18), results in lower damping estimates than using the exact formula, (A.13) - (A.17). This under-estimate of the damping is recovered (in part if not entirely) by the artificial widening of the spectral peaks caused by windowing the time domain data before computing the FFT's [Oppenheim 1979, Press 1988]. Using equation (5.11) or (A.18) for damping estimation from power spectra of acceleration data is contingent upon the following assumptions:

1. The damping is small (< 10%) and the damping mechanism is viscous. For damping < 2%, the truncation effects are insignificant.

2. The original discrete acceleration record can be represented by a Fourier series.
3. Each power spectrum coordinate equals the amplitude squared of the corresponding Fourier series term.
4. The response is narrow band, stationary, and steady-state and the structure behaves linearly.
5. Power spectrum computation does not appreciably widen the spectral peaks. This is often the over-riding assumption.

In summary, damping estimates from impulse responses are the most accurate, if an adequately strong and brief impulse can be generated. The absence of forced response experiments requires frequency domain estimates. These estimates are often upper bounds due to artificial peak widening in the process of computing averaged windowed power spectra.

APPENDIX B

POWER SPECTRA, PHASE SPECTRA, ESTIMATED FREQUENCIES, AND ESTIMATED DAMPING RATIOS: FIRST SET OF MEASUREMENTS

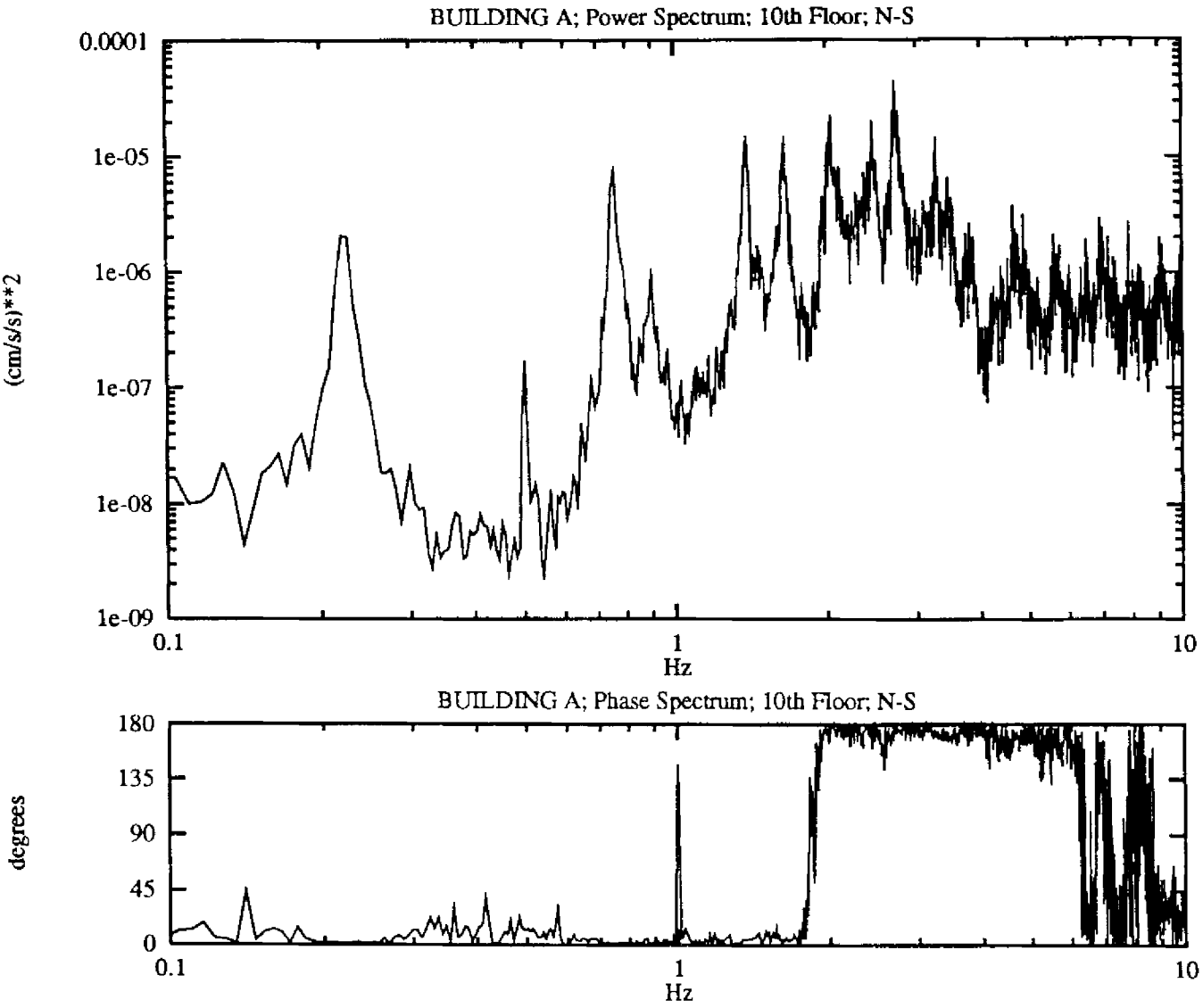
Parameters reported in Section 6 are estimated from the spectra shown in Appendix B. This data will serve as a base-line set of measurements for comparison to parameters obtained from measurements in stronger winds. The measurement dates are as follows:

Building A:	February 4, 1991
Building B:	February 7, 1991
Building C:	February 22, 1991
Building D:	February 22, 1991

FIGURE B-1

BUILDING A - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 4, 1991
Measurement	Sensor Location	10th Floor, Center
Reference	Sensor Location	20th Floor, Center
	Sensor Direction	North-South



Peaks In G10.20n

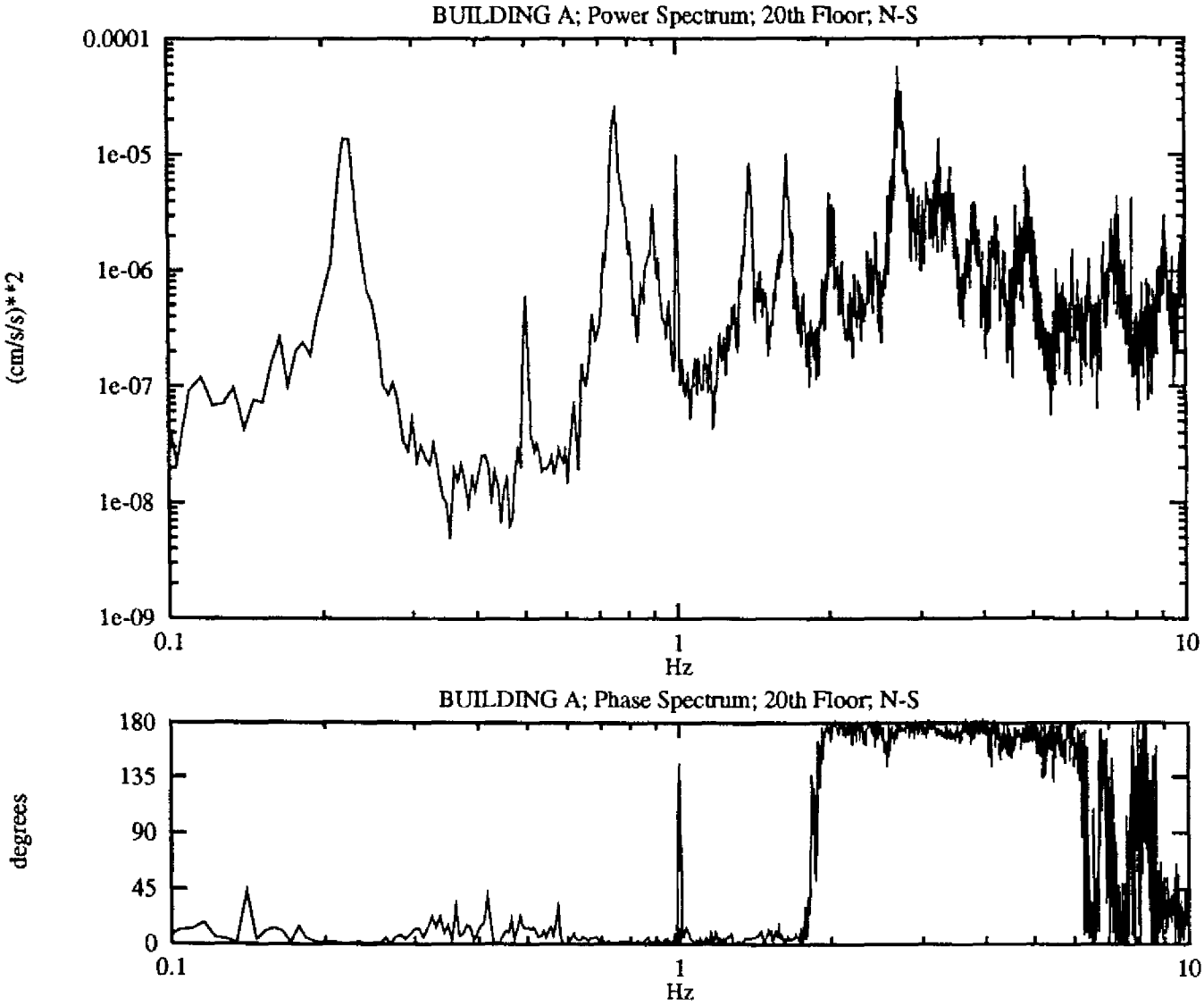
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2197 N1	0.2225	2.0383e-06	2.1789e-06	0	2.1
0.5005 T1	0.5003	1.6904e-07	1.6921e-07	13	0.7
0.7568 N2	0.7556	7.3875e-06	7.4916e-06	1	1.0
0.8972 T2	0.8966	1.0517e-06	1.0561e-06	2	0.8
1.3977 N3	1.3964	1.5605e-05	1.5914e-05	5	0.5
1.6541	1.6532	1.5315e-05	1.5557e-05	4	0.3
2.0447	2.0434	2.2421e-05	2.2650e-05	176	0.4
2.4658	2.4665	2.0156e-05	2.0266e-05	170	0.5
2.7283	2.7277	4.5020e-05	4.4823e-05	176	0.2
3.2837	3.2828	1.4461e-05	1.4782e-05	170	0.1

RMS-acc	RMS-vel	RMS-dsp
5.441e-02	3.752e-03	1.620e-03

FIGURE B-2

BUILDING A - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 4, 1991
Measurement	Sensor Location	20th Floor, Center
Reference	Sensor Location	10th Floor, Center
	Sensor Direction	North-South



Peaks In G20.10n

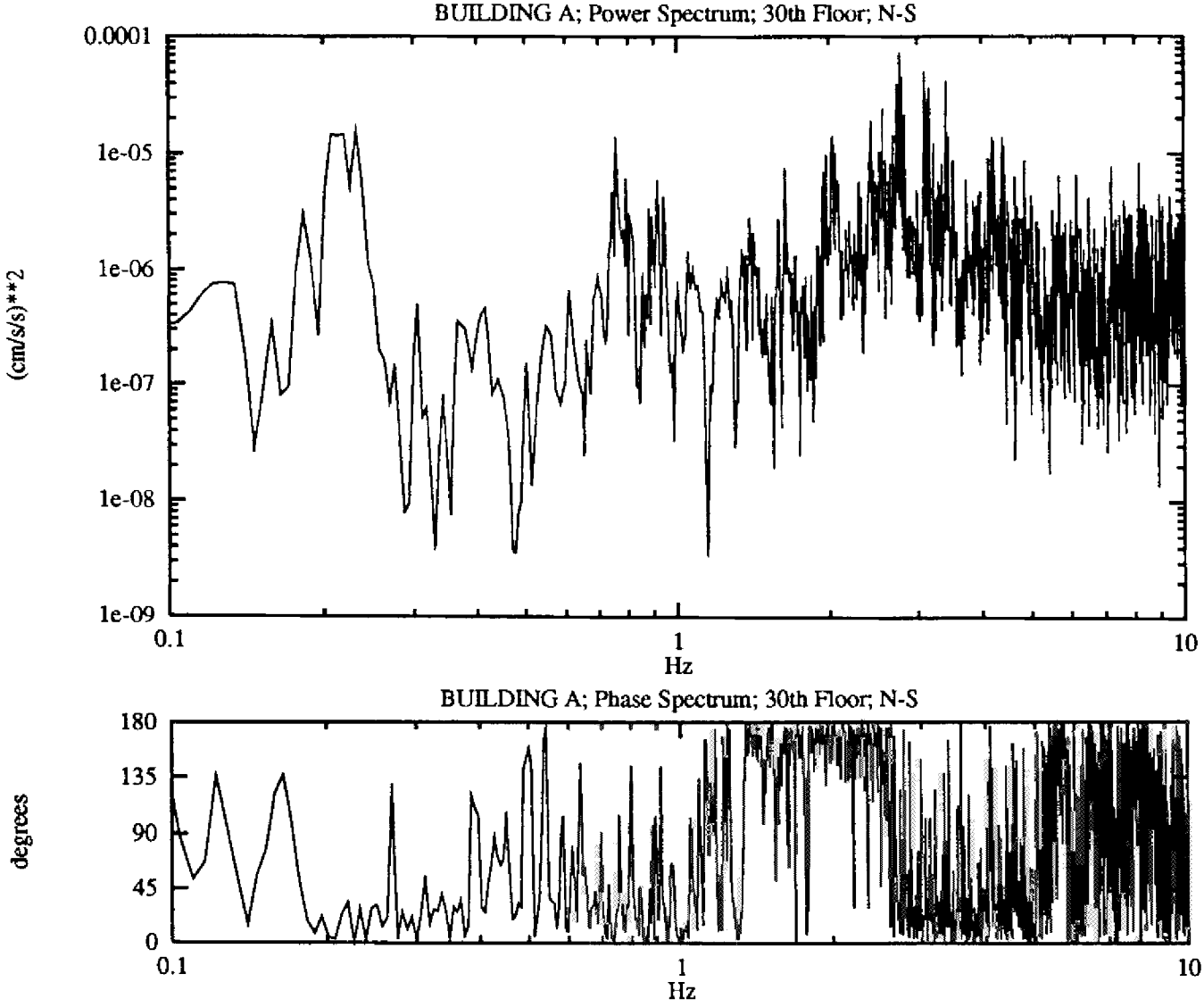
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2197 N1	0.2227	1.3643e-05	1.4678e-05	0	2.0
0.5005 T1	0.5003	5.9826e-07	5.9837e-07	13	0.8
0.7568 N2	0.7554	2.4299e-05	2.4699e-05	1	1.0
0.8972 T2	0.8968	3.7739e-06	3.7812e-06	2	0.8
1.0010	1.0005	9.8104e-06	9.8497e-06	147	0.3
1.3977 N3	1.3963	8.4339e-06	8.5831e-06	5	0.6
1.6541	1.6534	1.0178e-05	1.0252e-05	4	0.4
2.0020	2.0015	4.7157e-06	4.7535e-06	168	0.4
2.7283	2.7275	5.8616e-05	5.8174e-05	176	0.2
3.2837	3.2825	1.3784e-05	1.4305e-05	170	0.1
3.4607	3.4602	7.7546e-06	7.7486e-06	170	0.2

RMS-acc	RMS-vel	RMS-dsp
5.380e-02	6.087e-03	3.827e-03

FIGURE B-3

BUILDING A - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 4, 1991
Measurement	Sensor Location	30th Floor, Center
Reference	Sensor Location	10th Floor, Center
	Sensor Direction	North-South



Peaks In G30.10n

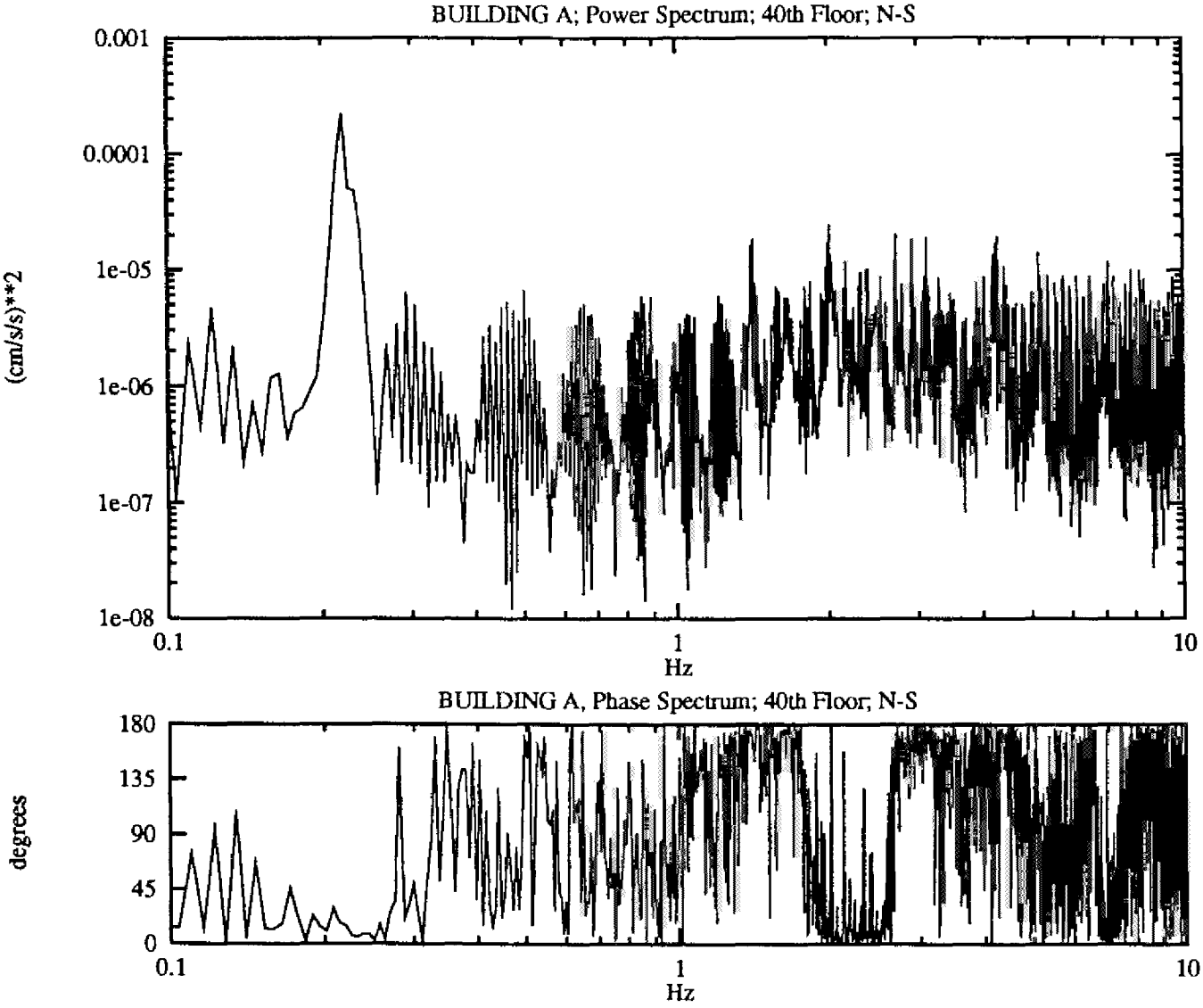
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2319 N1	0.2321	1.5934e-05	1.5941e-05	28	1.8
0.7568 N2	0.7575	1.3971e-05	1.4096e-05	105	0.6
0.9155	0.9152	5.8761e-06	5.8860e-06	144	0.4
1.6357	1.6357	7.6040e-06	7.6294e-06	63	0.1
1.9775	1.9775	9.6823e-06	9.6560e-06	146	0.1
2.7710	2.7719	7.3553e-05	7.5340e-05	173	0.2
3.1006	3.0998	5.0418e-05	5.0545e-05	36	0.1

RMS-acc	RMS-vel	RMS-dsp
6.615e-02	8.571e-03	6.473e-03

FIGURE B-4

BUILDING A - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 4, 1991
Measurement	Sensor Location	40th Floor, Center
Reference	Sensor Location	10th Floor, Center
	Sensor Direction	North-South



Peaks In G40.10n

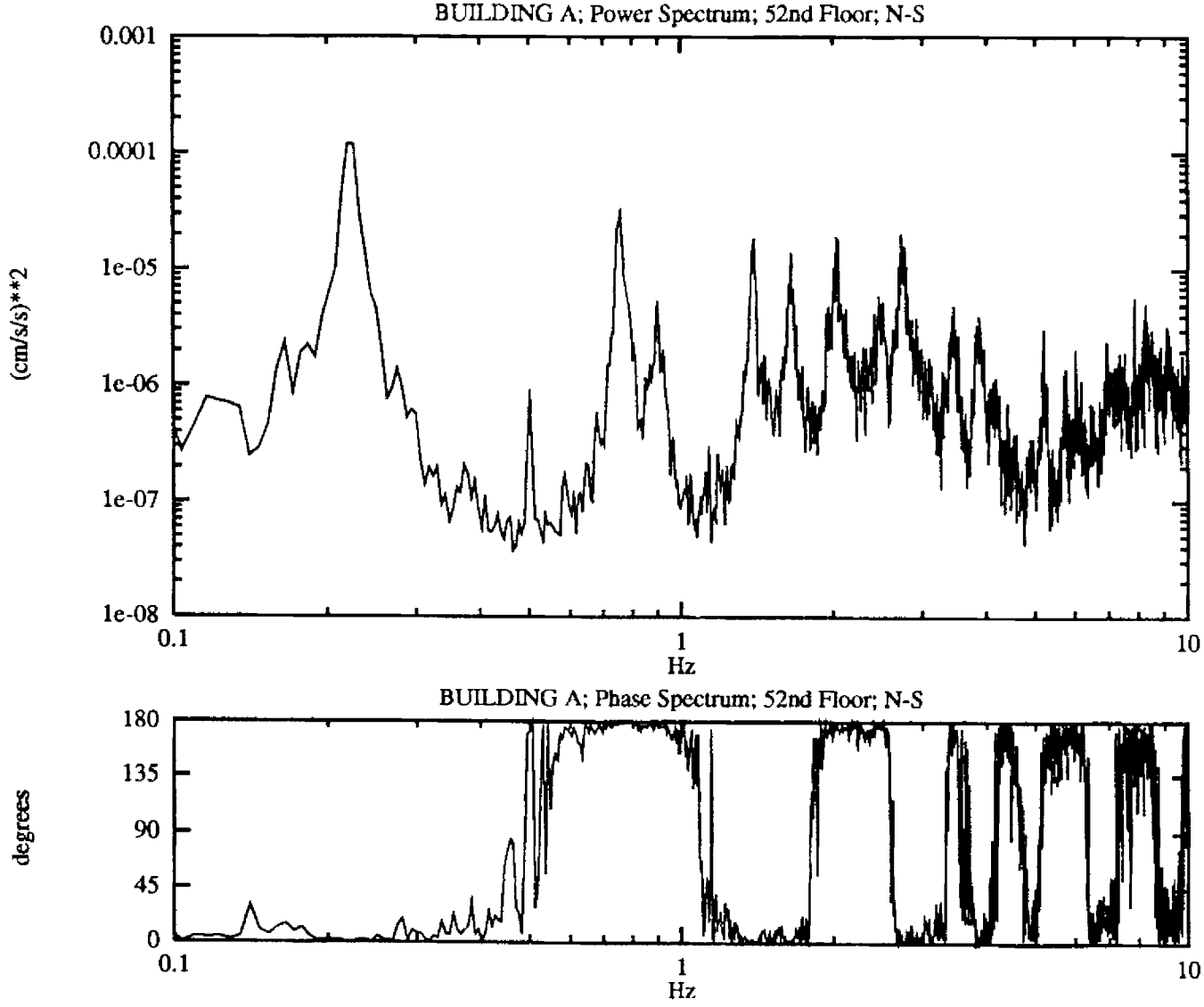
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2197 N1	0.2194	2.2120e-04	2.2162e-04	15	1.5
1.4160	1.4160	1.8379e-05	1.8299e-05	179	0.1
2.0142	2.0140	2.4589e-05	2.4557e-05	5	0.2

RMS-acc	RMS-vel	RMS-dsp
8.417e-02	1.721e-02	1.272e-02

FIGURE B-5

BUILDING A - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 4, 1991
Measurement	Sensor Location	52nd Floor, Center
Reference	Sensor Location	10th Floor, Center
	Sensor Direction	North-South



Peaks In G52.10n

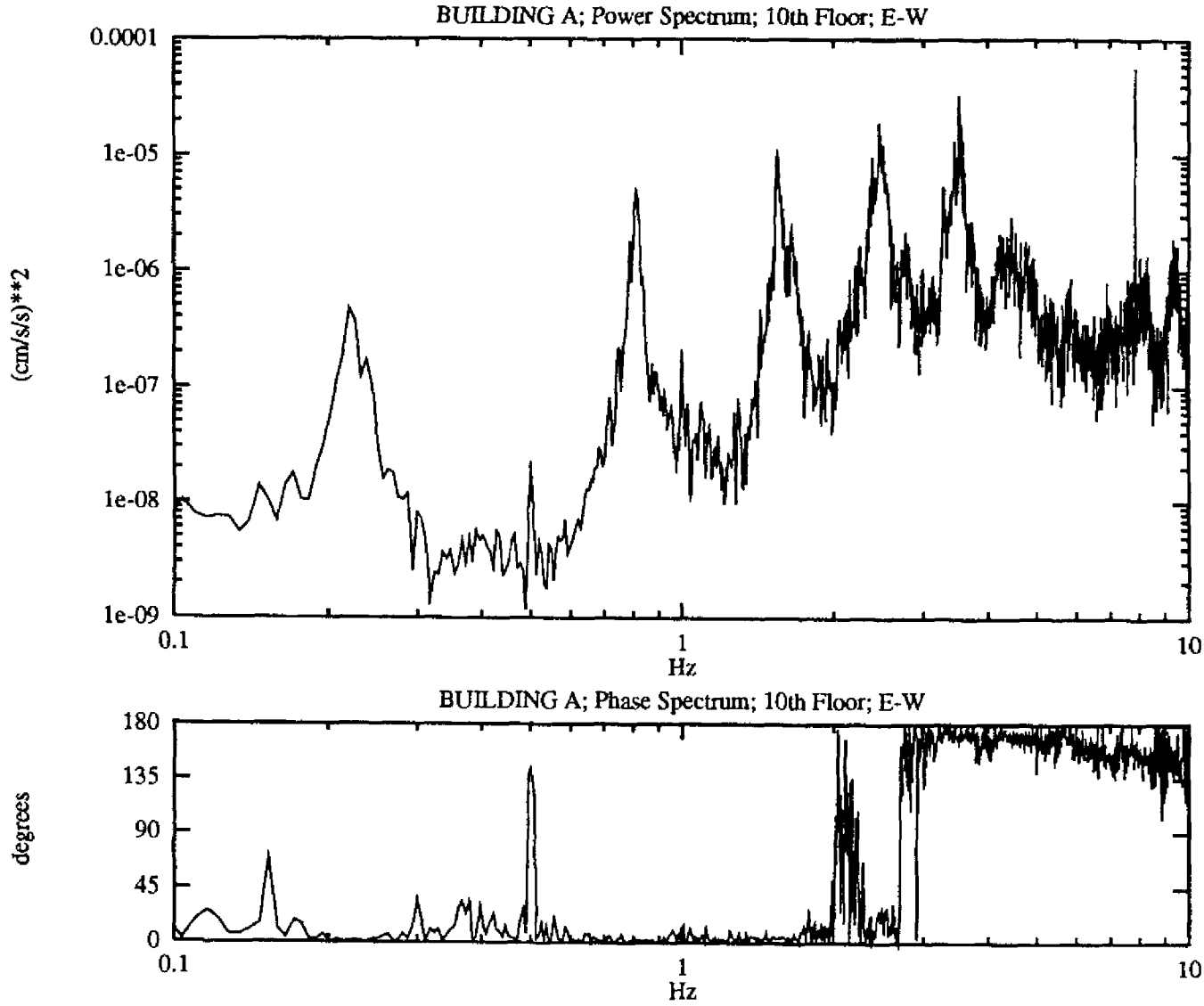
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2197 N1	0.2227	1.1880e-04	1.2786e-04	0	2.0
0.5005 T1	0.5004	9.1953e-07	9.1968e-07	172	0.7
0.7568 N2	0.7555	3.0521e-05	3.0920e-05	179	1.0
0.8972 T2	0.8971	5.3543e-06	5.3532e-06	179	0.8
1.3977 N3	1.3959	1.8528e-05	1.9073e-05	1	0.6
1.6541	1.6535	1.3813e-05	1.3947e-05	3	0.3
2.0325	2.0330	1.9264e-05	1.9312e-05	176	0.8
2.4658	2.4676	5.8860e-06	6.0052e-06	176	0.7
2.7283	2.7274	2.0158e-05	2.0146e-05	4	0.2
3.4607	3.4607	4.8732e-06	5.0068e-06	163	0.1
3.8818	3.8817	4.0096e-06	3.9339e-06	6	0.1

RMS-acc	RMS-vel	RMS-dsp
5.674e-02	1.486e-02	1.078e-02

FIGURE B-6

BUILDING A - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 4, 1991
Measurement	Sensor Location	10th Floor, Center
Reference	Sensor Location	20th Floor, Center
	Sensor Direction	East-West



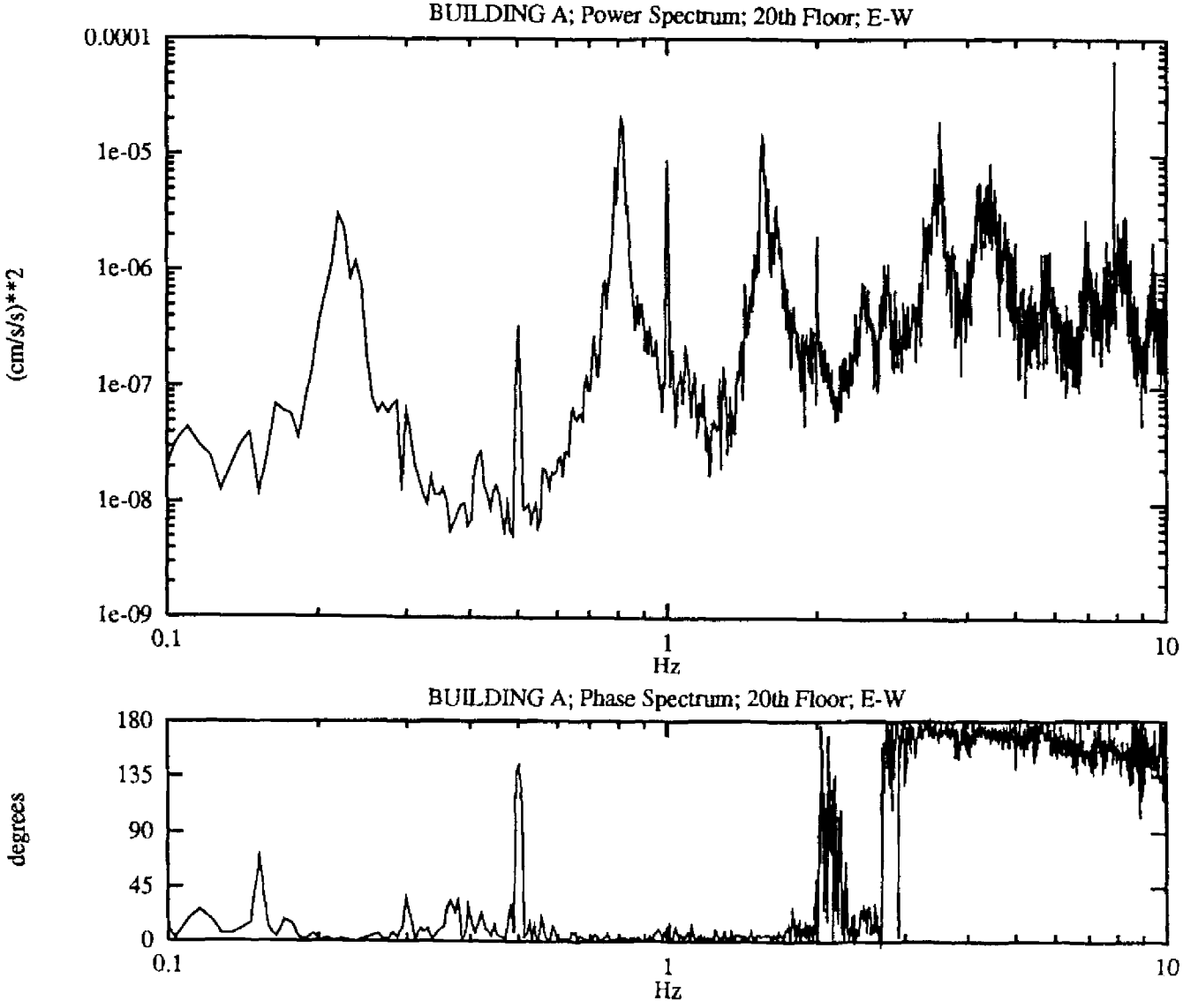
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2197	0.2211	4.7275e-07	4.8324e-07	1	2.5
0.5005	0.5002	2.2547e-08	2.2577e-08	143	0.7
0.8118	0.8127	4.8938e-06	4.9155e-06	3	1.0
1.5503	1.5512	1.0232e-05	1.0259e-05	4	0.7
2.4658	2.4670	1.9098e-05	1.9372e-05	23	0.5
3.5217	3.5213	3.3054e-05	3.2663e-05	171	0.3

RMS-acc	RMS-vel	RMS-dsp
4.156e-02	2.618e-03	1.233e-03

FIGURE B-7

BUILDING A - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 4, 1991
Measurement	Sensor Location	20th Floor, Center
Reference	Sensor Location	20th Floor, Center
	Sensor Direction	East-West



Peaks In G20.10e

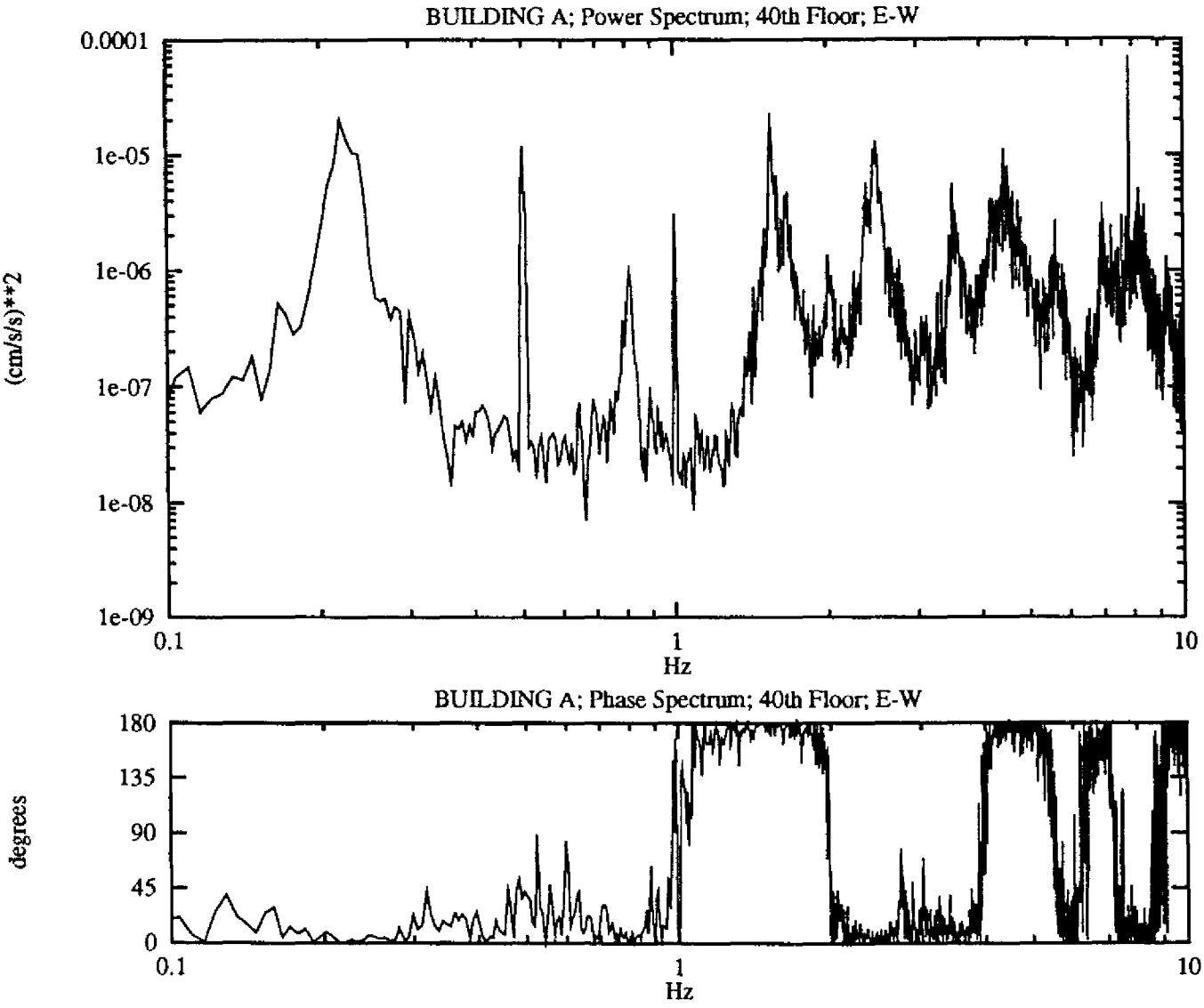
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2197	0.2210	3.0824e-06	3.1450e-06	1	2.7
0.5005	0.5003	3.3415e-07	3.3434e-07	143	0.7
0.8118	0.8127	2.0372e-05	2.0452e-05	3	0.9
0.9216	0.9231	3.0714e-07	3.1129e-07	4	1.3
1.5503	1.5519	1.4445e-05	1.4566e-05	4	0.7
3.5217	3.5211	1.9257e-05	1.8835e-05	171	0.2

RMS-acc	RMS-vel	RMS-dsp
4.255e-02	3.975e-03	2.341e-03

FIGURE B-8

BUILDING A - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 4, 1991
Measurement	Sensor Location	40th Floor, Center
Reference	Sensor Location	10th Floor, Center
	Sensor Direction	East-West



Peaks In G40.10e

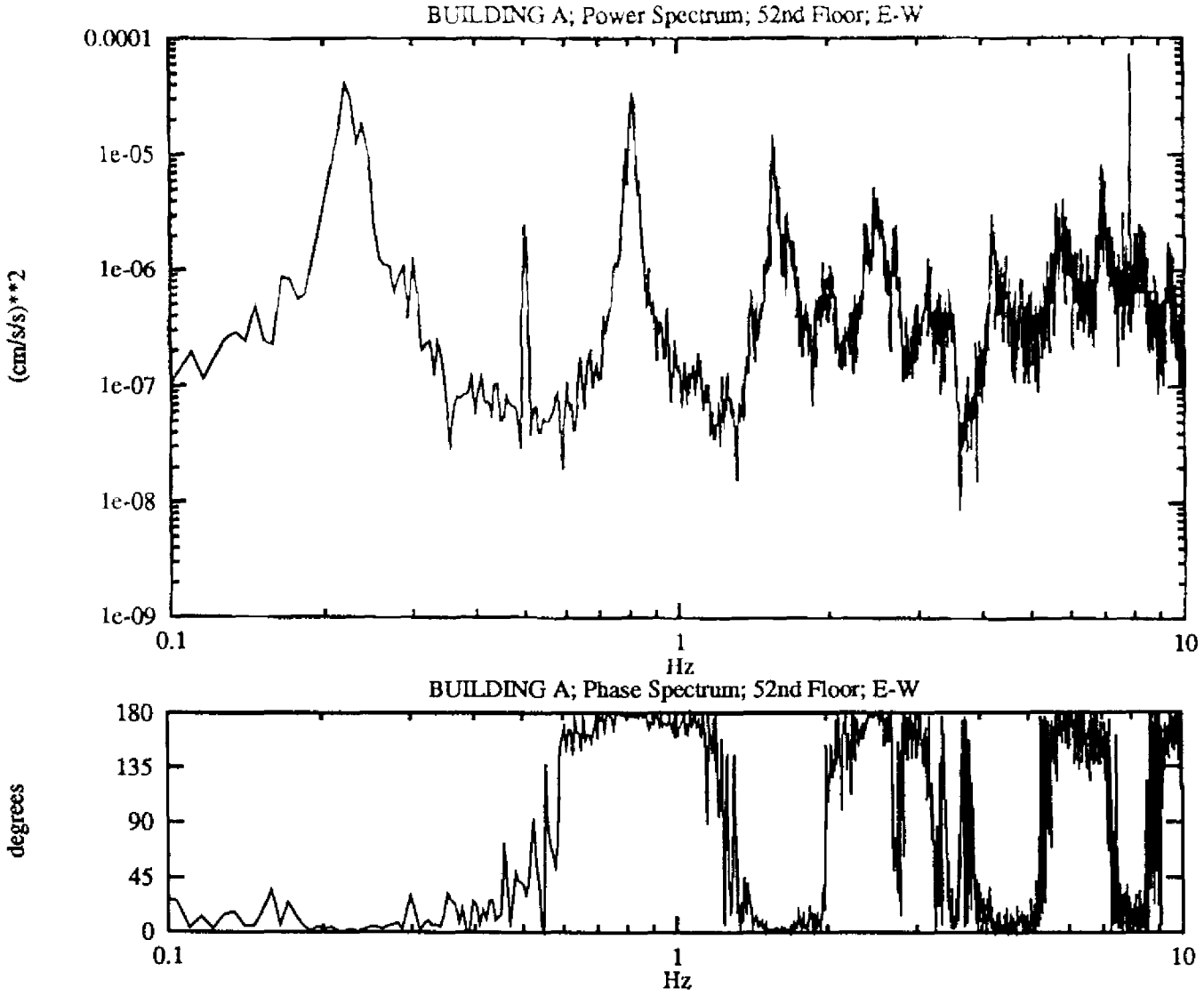
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2197	0.2206	2.0189e-05	2.0377e-05	1	2.9
0.5005	0.5004	1.1825e-05	1.1828e-05	38	0.6
0.8118	0.8117	1.0042e-06	1.0040e-06	0	1.0
1.5503	1.5498	2.3042e-05	2.3127e-05	170	0.5
2.4902	2.4898	1.3338e-05	1.3351e-05	5	1.0
3.5217	3.5207	5.6267e-06	5.6326e-06	5	0.3
4.4556	4.4550	1.1168e-05	1.1206e-05	177	0.2

RMS-acc	RMS-vel	RMS-dsp
4.503e-02	7.520e-03	5.459e-03

FIGURE B-9

BUILDING A - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 4, 1991
Measurement	Sensor Location	52nd Floor, Center
Reference	Sensor Location	10th Floor, Center
	Sensor Direction	East-West



Peaks In G52.10e

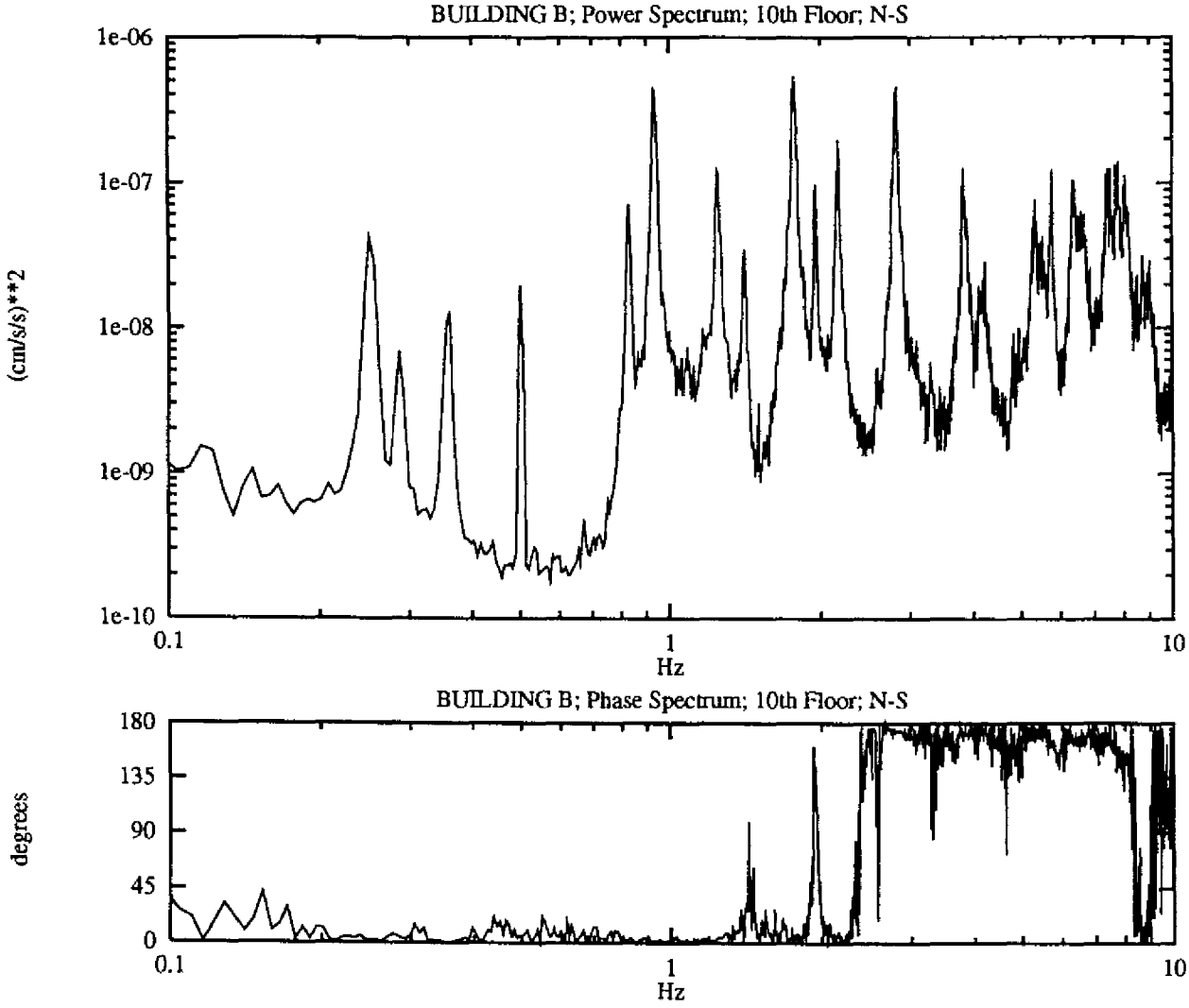
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2197	0.2211	3.9923e-05	4.0730e-05	0	2.9
0.5005	0.5003	2.5103e-06	2.5127e-06	39	0.7
0.8118	0.8129	3.1305e-05	3.1509e-05	177	0.9
1.5503	1.5508	1.4744e-05	1.4767e-05	2	0.7
2.4658	2.4663	5.2409e-06	5.2899e-06	177	0.5

RMS-acc	RMS-vel	RMS-dsp
4.274e-02	9.882e-03	7.046e-03

FIGURE B-10

BUILDING B - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 7, 1991
Measurement	Sensor Location	10th Floor, Center
Reference	Sensor Location	20th Floor, Center
	Sensor Direction	North-South



Peaks In G10.20n

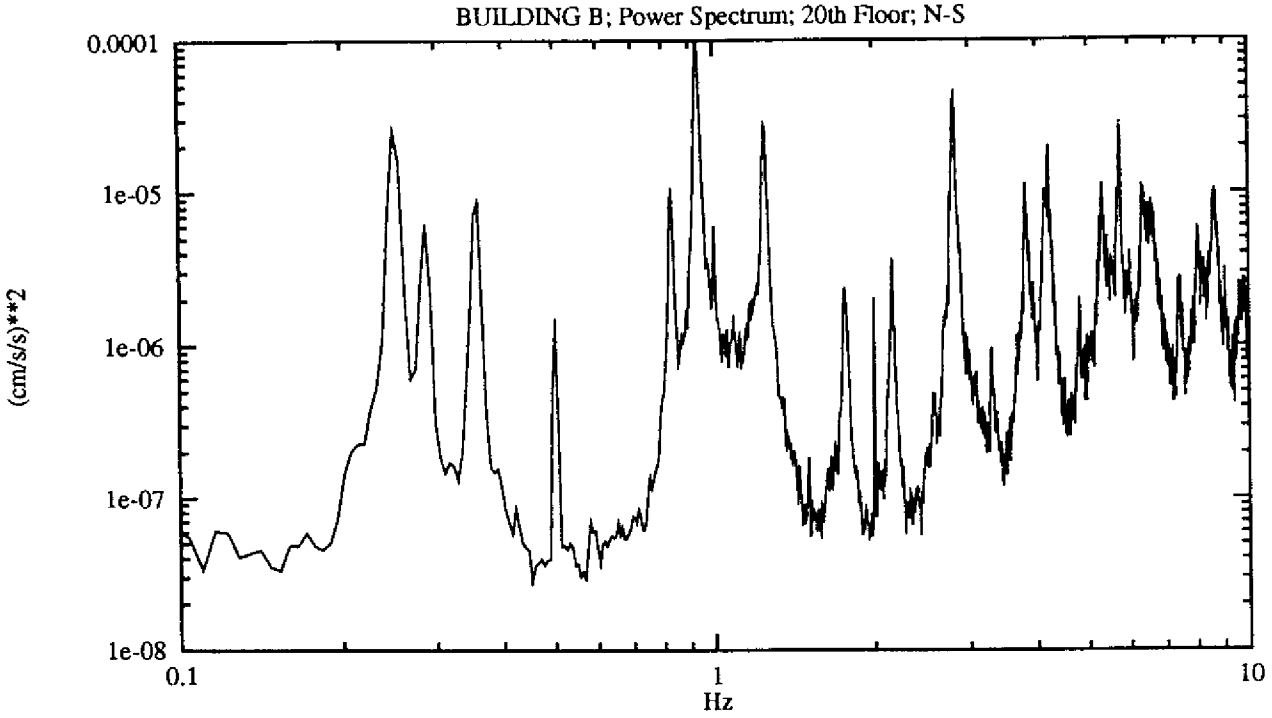
Hz-data		Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2502		0.2513	4.2749e-08	4.3379e-08	2	2.4
0.2869	T1	0.2868	6.8865e-09	6.8867e-09	4	1.7
0.3601		0.3581	1.2578e-08	1.3211e-08	0	1.1
0.5005		0.5002	1.9753e-08	1.9776e-08	6	0.6
0.8301	T2	0.8290	7.1446e-08	7.2032e-08	3	0.9
0.9277	T3	0.9304	4.4073e-07	4.6869e-07	2	1.0
1.2451		1.2474	1.2162e-07	1.2538e-07	5	0.7
1.4099	T4	1.4102	3.5103e-08	3.5187e-08	33	0.8
1.7700		1.7685	5.0356e-07	5.0757e-07	3	0.9
1.9531	T5	1.9518	9.7993e-08	9.8953e-08	93	0.5
2.1667		2.1655	1.9735e-07	1.9884e-07	3	0.5
2.8259		2.8246	4.6336e-07	4.6566e-07	173	0.5
3.8208		3.8215	1.2677e-07	1.2526e-07	170	0.5
3.9490		3.9483	2.0414e-08	2.0140e-08	176	0.6
4.2297		4.2301	2.9024e-08	2.8871e-08	172	0.2

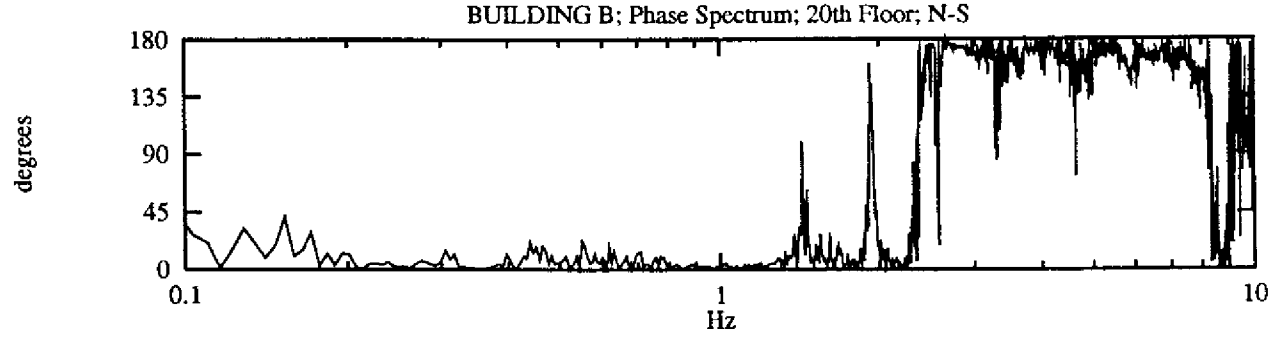
RMS-acc	RMS-vel	RMS-dsp
6.440e-03	5.138e-04	2.807e-04

FIGURE B-11

BUILDING B - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 7, 1991
Measurement	Sensor Location	20th Floor, Center
Reference	Sensor Location	10th Floor, Center
	Sensor Direction	North-South





Peaks In G20.10n

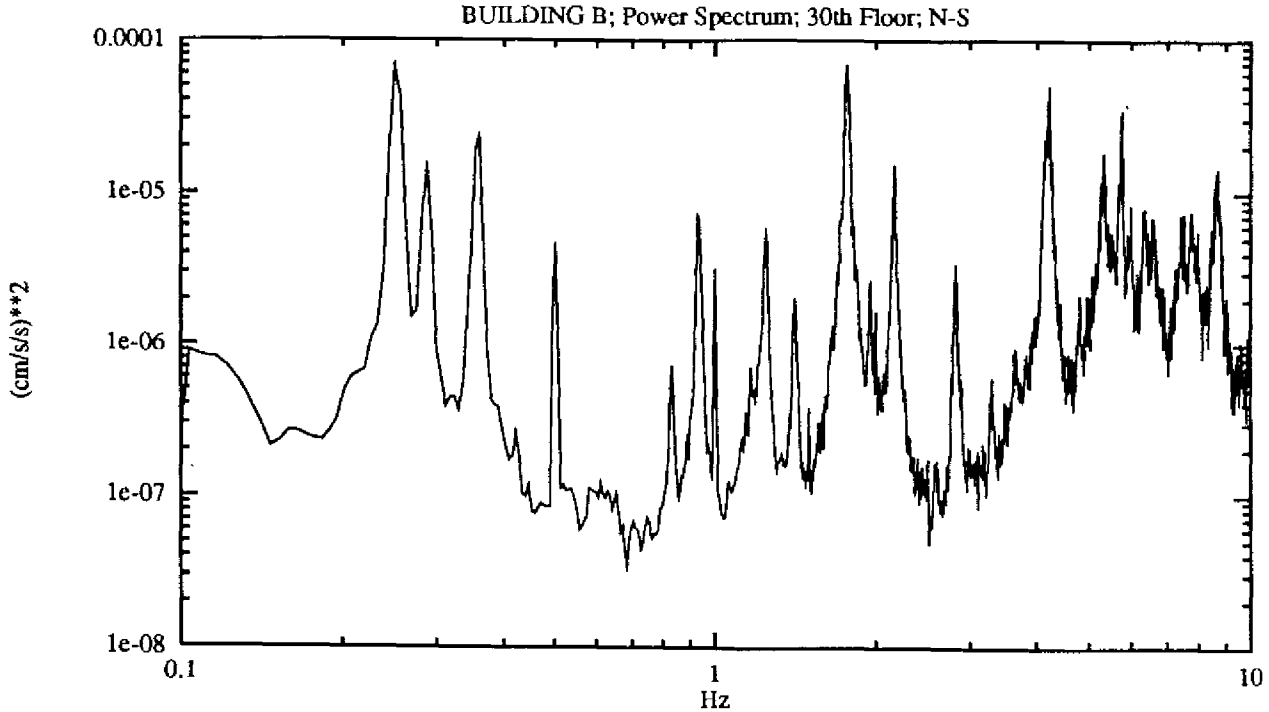
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2502	0.2512	2.5807e-05	2.6146e-05	2	2.4
0.2869 T1	0.2868	6.3026e-06	6.3032e-06	4	1.5
0.3601	0.3582	8.7874e-06	9.2266e-06	0	1.0
0.5005	0.5003	1.5105e-06	1.5125e-06	6	0.7
0.8301 T2	0.8291	1.0673e-05	1.0759e-05	3	0.9
0.9277 T3	0.9305	9.8234e-05	1.0461e-04	2	1.0
1.0010	1.0006	6.0538e-06	6.0722e-06	1	0.4
1.2451	1.2471	2.7618e-05	2.8193e-05	5	0.7
1.7639	1.7656	2.3443e-06	2.3525e-06	3	1.1
2.0020	2.0008	2.0530e-06	2.0713e-06	18	0.1
2.1667	2.1657	3.7088e-06	3.7327e-06	3	0.5
2.8259	2.8243	4.6485e-05	4.6968e-05	173	0.5
3.3020 T6	3.3001	9.6800e-07	9.8720e-07	130	0.3
3.8208	3.8216	1.1379e-05	1.1444e-05	170	0.4
3.9490	3.9485	2.5657e-06	2.5779e-06	176	0.9
4.2297	4.2305	2.0037e-05	2.0742e-05	172	0.2

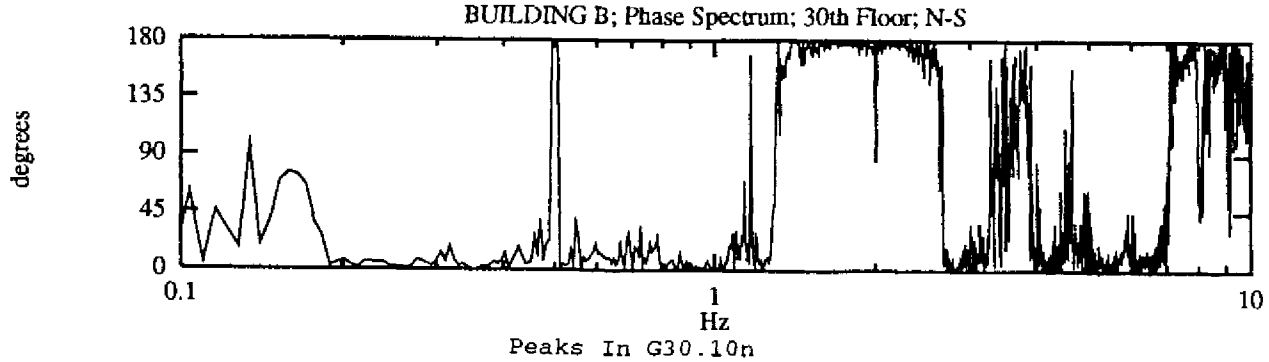
RMS-acc	RMS-vel	RMS-dsp
6.756e-02	7.412e-03	3.940e-03

FIGURE B-12

BUILDING B - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 7, 1991
Measurement	Sensor Location	30th Floor, Center
Reference	Sensor Location	10th Floor, Center
	Sensor Direction	North-South





Peaks In G30.10n

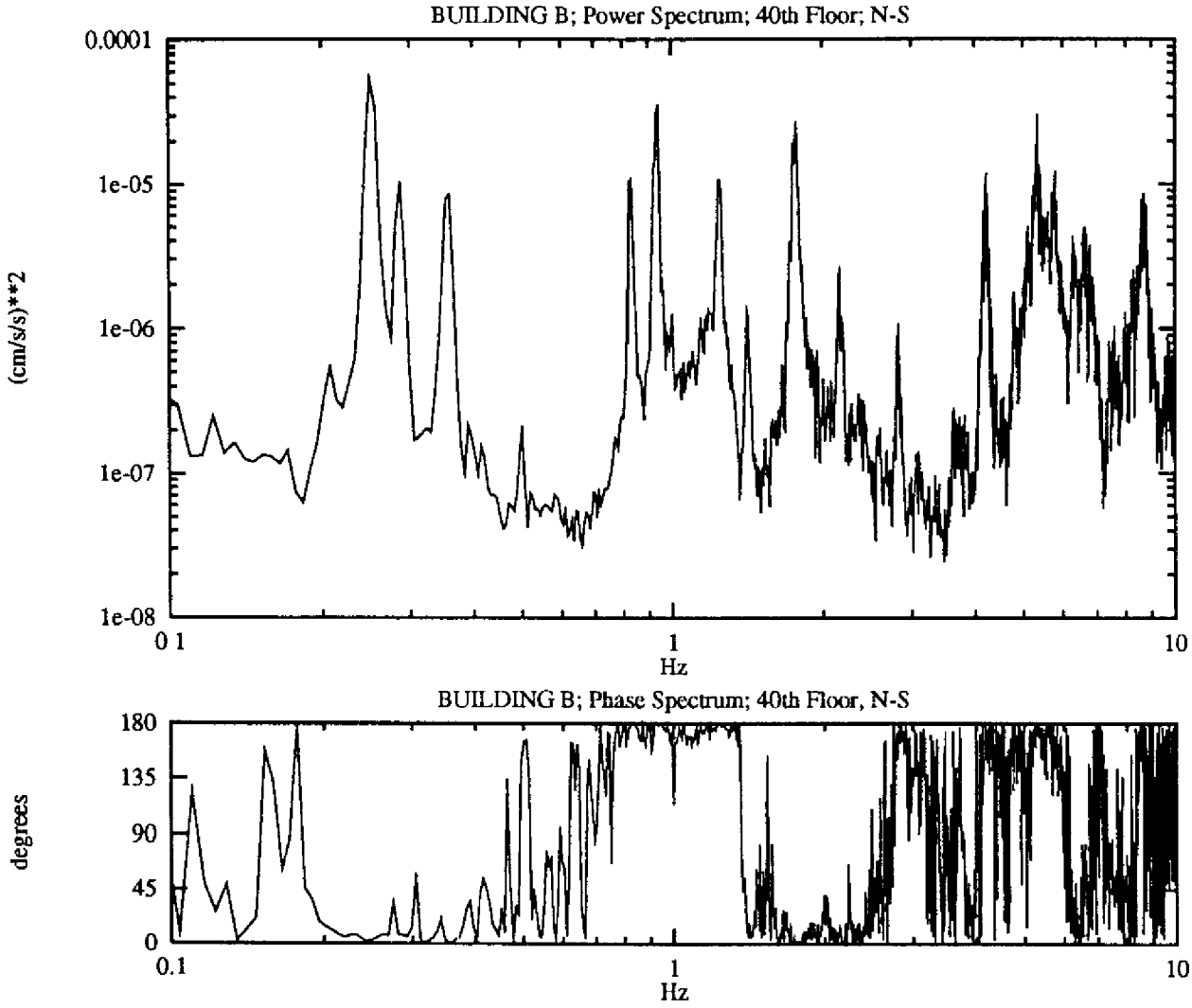
Hz-data		Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2502		0.2512	6.6898e-05	6.7841e-05	2	2.4
0.2869	T1	0.2868	1.5822e-05	1.5823e-05	5	1.5
0.3601		0.3581	2.3460e-05	2.4650e-05	1	1.0
0.5005		0.5002	4.7395e-06	4.7460e-06	179	0.7
0.8301	T2	0.8292	7.3327e-07	7.3761e-07	8	0.9
0.9277	T3	0.9303	7.1502e-06	7.5772e-06	2	1.0
1.0010		1.0004	3.1658e-06	3.1888e-06	2	0.3
1.2451		1.2461	5.4706e-06	5.4948e-06	8	0.7
1.4099	T4	1.4102	2.0440e-06	2.0452e-06	179	0.8
1.7700		1.7684	6.4706e-05	6.5267e-05	177	0.9
1.9531	T5	1.9509	2.5350e-06	2.6040e-06	176	0.6
2.1667		2.1655	1.5456e-05	1.5646e-05	178	0.5
2.8259		2.8247	3.4983e-06	3.5167e-06	2	0.5
4.2297		4.2306	4.9969e-05	5.0545e-05	2	0.2
5.3284		5.3288	1.8501e-05	1.8835e-05	2	0.3
5.7556		5.7557	3.4598e-05	3.4332e-05	8	0.4

RMS-acc	RMS-vel	RMS-dsp
7.292e-02	1.007e-01	6.318e-03

FIGURE B-13

BUILDING B - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 7, 1991
Measurement	Sensor Location	40th Floor, Center
Reference	Sensor Location	10th Floor, Center
	Sensor Direction	North-South



Peaks In G40.10n

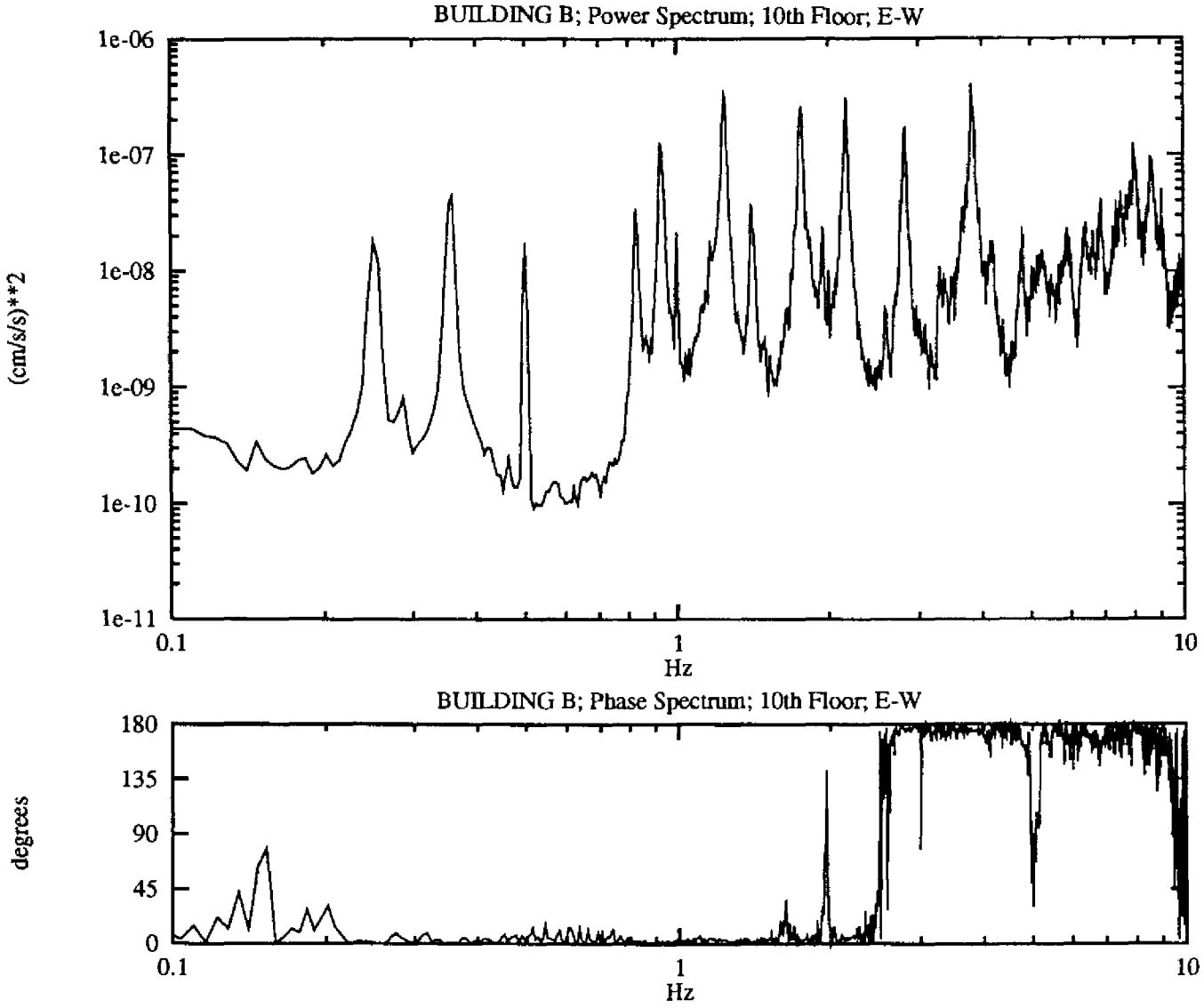
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2502	0.2512	5.4492e-05	5.5306e-05	3	2.3
0.2869 T1	0.2864	1.0573e-05	1.0610e-05	7	1.4
0.3601	0.3575	8.4301e-06	9.0357e-06	1	1.3
0.5005	0.4997	2.1479e-07	2.1647e-07	165	1.0
0.8301 T2	0.8275	1.0780e-05	1.1347e-05	179	0 9
0.9399 T3	0.9375	3.3593e-05	3.5822e-05	179	0.5
1.0010	1.0000	1.2520e-06	1.2666e-06	114	0.6
1.2451	1.2452	1.0732e-05	1.0732e-05	179	0.8
1.4099 T4	1.4120	1.3258e-06	1.3672e-06	17	0.8
1.7761	1.7749	2.7003e-05	2.7299e-05	2	0.5
2.1667	2.1640	2.4450e-06	2.6226e-06	0	0.3
2.8259	2.8246	1.0786e-06	1.0878e-06	169	0.3
4.2358	4.2331	1.1243e-05	1.1802e-05	177	0.3
5.3528	5.3518	3.0246e-05	3.1471e-05	174	0.1
5.8044	5.8030	1.2482e-05	1.2875e-05	178	0.2

RMS-acc	RMS-vel	RMS-dsp
5.206e-02	8.614e-03	5.405e-03

FIGURE B-14

BUILDING B - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 7, 1991
Measurement	Sensor Location	10th Floor, Center
Reference	Sensor Location	20th Floor, Center
	Sensor Direction	East-West



Peaks In G10.20e

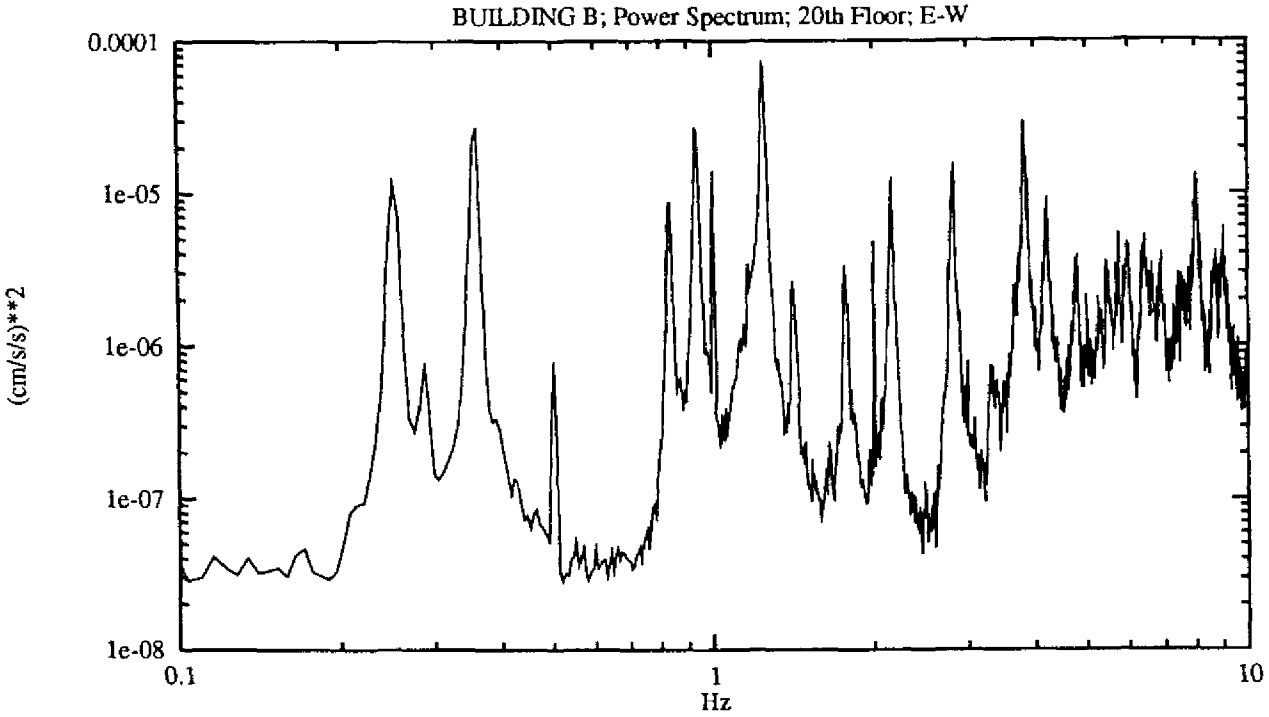
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2502	0.2512	1.8296e-08	1.8515e-08	0	2.4
0.3601	0.3581	4.4522e-08	4.6886e-08	1	1.0
0.5005	0.5002	1.7424e-08	1.7455e-08	5	0.6
0.8301 T2	0.8294	3.4397e-08	3.4517e-08	3	0.9
0.9277 T3	0.9303	1.2379e-07	1.3108e-07	2	1.0
1.0010	1.0006	2.1840e-08	2.1915e-08	0	0.3
1.2451	1.2469	3.3531e-07	3.4040e-07	2	0.7
1.4099 T4	1.4124	3.6174e-08	3.7195e-08	0	0.8
1.7700	1.7693	2.6096e-07	2.6147e-07	3	0.8
2.1667	2.1650	3.0756e-07	3.1199e-07	5	0.5
2.8259	2.8245	1.7293e-07	1.7416e-07	175	0.5
3.8208	3.8210	4.0677e-07	4.0419e-07	174	0.4

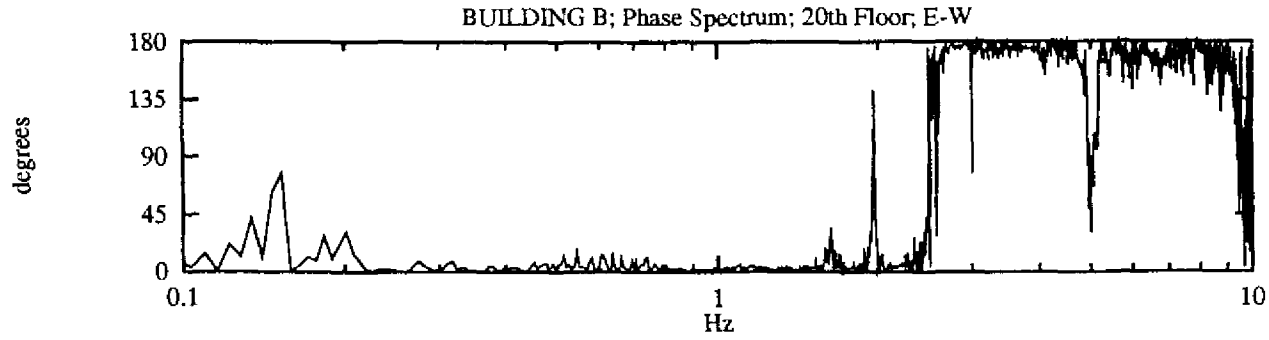
RMS-acc	RMS-vel	RMS-dsp
5.927e-03	4.261e-04	2.158e-04

FIGURE B-15

BUILDING B - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 7, 1991
Measurement	Sensor Location	20th Floor, Center
Reference	Sensor Location	10th Floor, Center
	Sensor Direction	East-West





Peaks In G20.10e

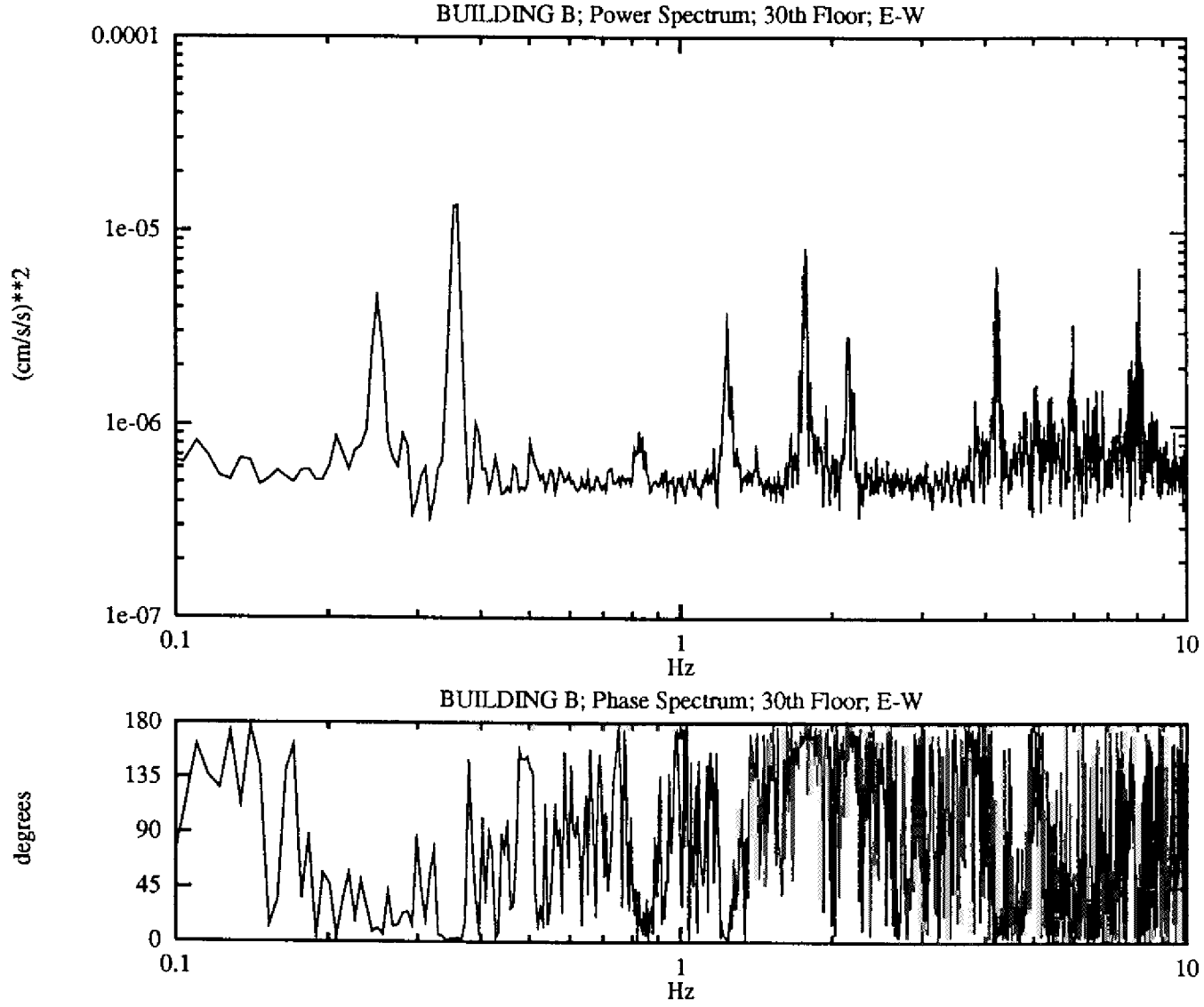
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2502	0.2512	1.1862e-05	1.2006e-05	0	2.3
0.3601	0.3580	2.6226e-05	2.7731e-05	1	1.0
0.5005	0.5002	8.0354e-07	8.0466e-07	5	0.7
0.8301 T2	0.8294	8.7261e-06	8.7582e-06	3	0.9
0.9277 T3	0.9303	2.6870e-05	2.8476e-05	2	1.0
1.0010	1.0005	1.3890e-05	1.3977e-05	0	0.3
1.2451	1.2470	6.9901e-05	7.1168e-05	2	0.7
1.4160 T4	1.4130	2.6314e-06	2.7325e-06	1	0.9
1.7700	1.7696	3.3644e-06	3.3714e-06	3	0.8
2.0020	2.0006	4.8394e-06	4.9472e-06	4	0.1
2.1667	2.1648	1.2643e-05	1.2964e-05	5	0.5
2.8259	2.8245	1.5691e-05	1.5736e-05	175	0.5
3.8208	3.8209	2.9261e-05	2.9325e-05	174	0.4
4.2297	4.2321	9.2176e-06	9.5367e-06	176	0.3

RMS-acc	RMS-vel	RMS-dsp
6.011e-02	6.332e-03	2.996e-03

FIGURE B-16

BUILDING B - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 7, 1991
Measurement	Sensor Location	30th Floor, Center
Reference	Sensor Location	10th Floor, Center
	Sensor Direction	East-West



Peaks In G30.10e

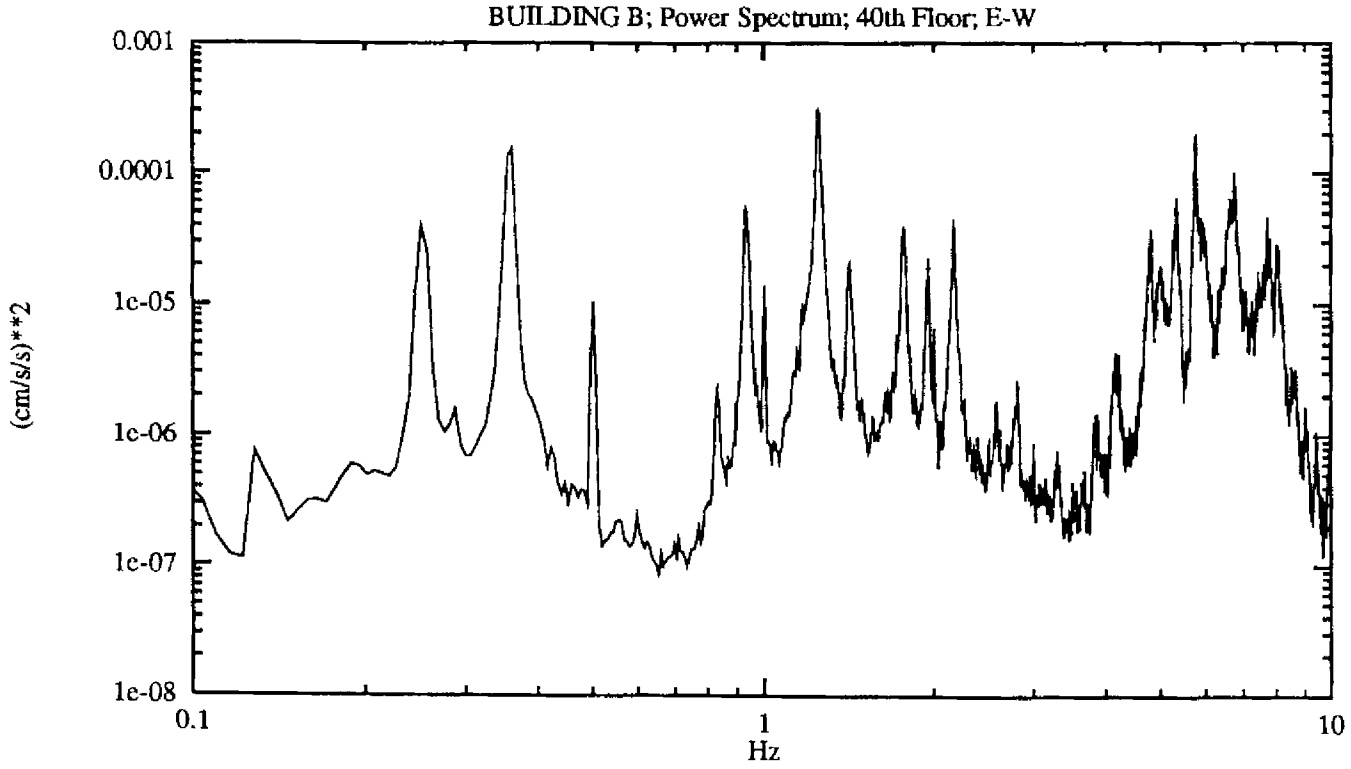
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2502	0.2503	4.4746e-06	4.4745e-06	11	2.4
0.3601	0.3571	1.3679e-05	1.4858e-05	4	1.4
0.8240 T2	0.8244	9.3133e-07	9.3202e-07	42	2.6
1.2390	1.2389	3.7996e-06	3.7923e-06	3	0.5
1.7700	1.7713	8.1924e-06	8.2552e-06	179	0.8
2.1423	2.1443	2.8611e-06	2.9057e-06	176	1.0

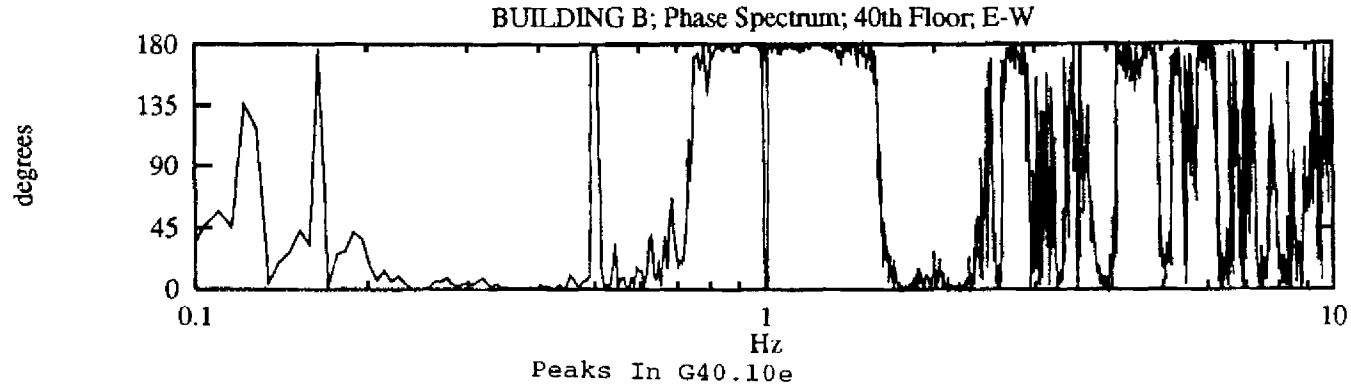
RMS-acc	RMS-vel	RMS-dsp
3.998e-02	7.270e-03	5.872e-03

FIGURE B-17

BUILDING B - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 7, 1991
Measurement	Sensor Location	40th Floor, Center
Reference	Sensor Location	10th Floor, Center
	Sensor Direction	East-West





Peaks In G40.10e

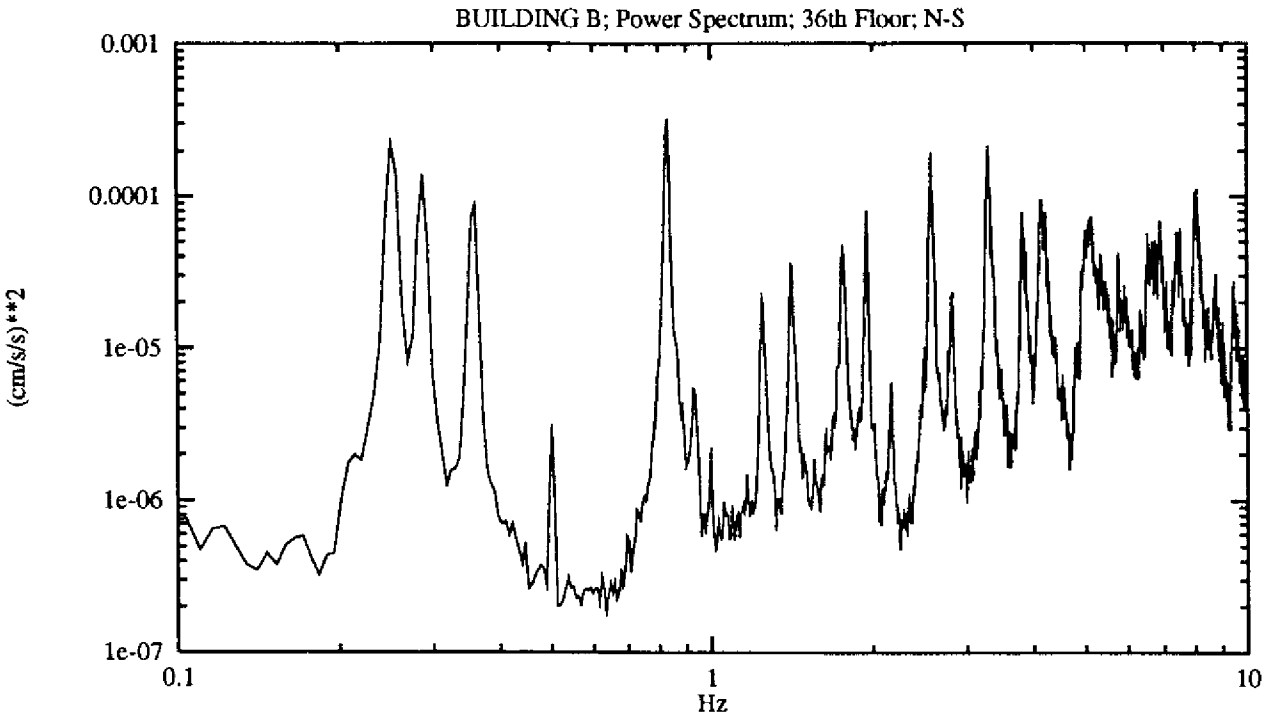
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2502	0.2512	4.0316e-05	4.0822e-05	0	2.4
0.3601	0.3579	1.5520e-04	1.6442e-04	0	1.0
0.5005	0.5002	1.0364e-05	1.0379e-05	176	0.6
0.8301 T2	0.8294	2.2624e-06	2.2687e-06	177	1.0
0.9277 T3	0.9303	5.3396e-05	5.6475e-05	180	1.0
1.0010	1.0004	1.3866e-05	1.3977e-05	2	0.3
1.2451	1.2471	3.0377e-04	3.0994e-04	179	0.7
1.4160 T4	1.4135	2.0855e-05	2.1517e-05	177	0.8
1.7700	1.7689	3.9637e-05	3.9935e-05	2	0.8
1.9531 T5	1.9522	2.2500e-05	2.2650e-05	8	0.6
2.1667	2.1655	4.5414e-05	4.5776e-05	1	0.5

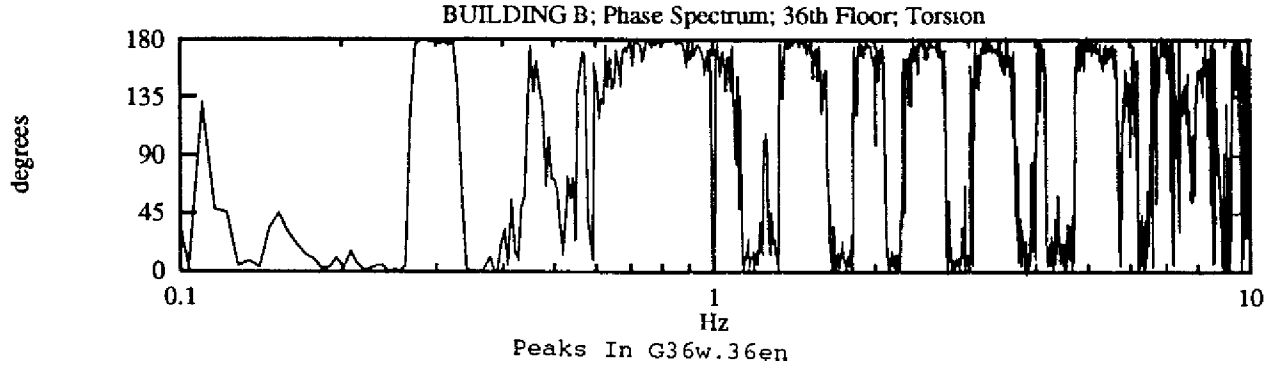
RMS-acc	RMS-vel	RMS-dsp
1.322e-01	1.333e-02	6.581e-03

FIGURE B-18

BUILDING B - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 7, 1991
Measurement	Sensor Location	36th Floor, East
Reference	Sensor Location	36th Floor, West
	Sensor Direction	North-South





Peaks In G36w.36en

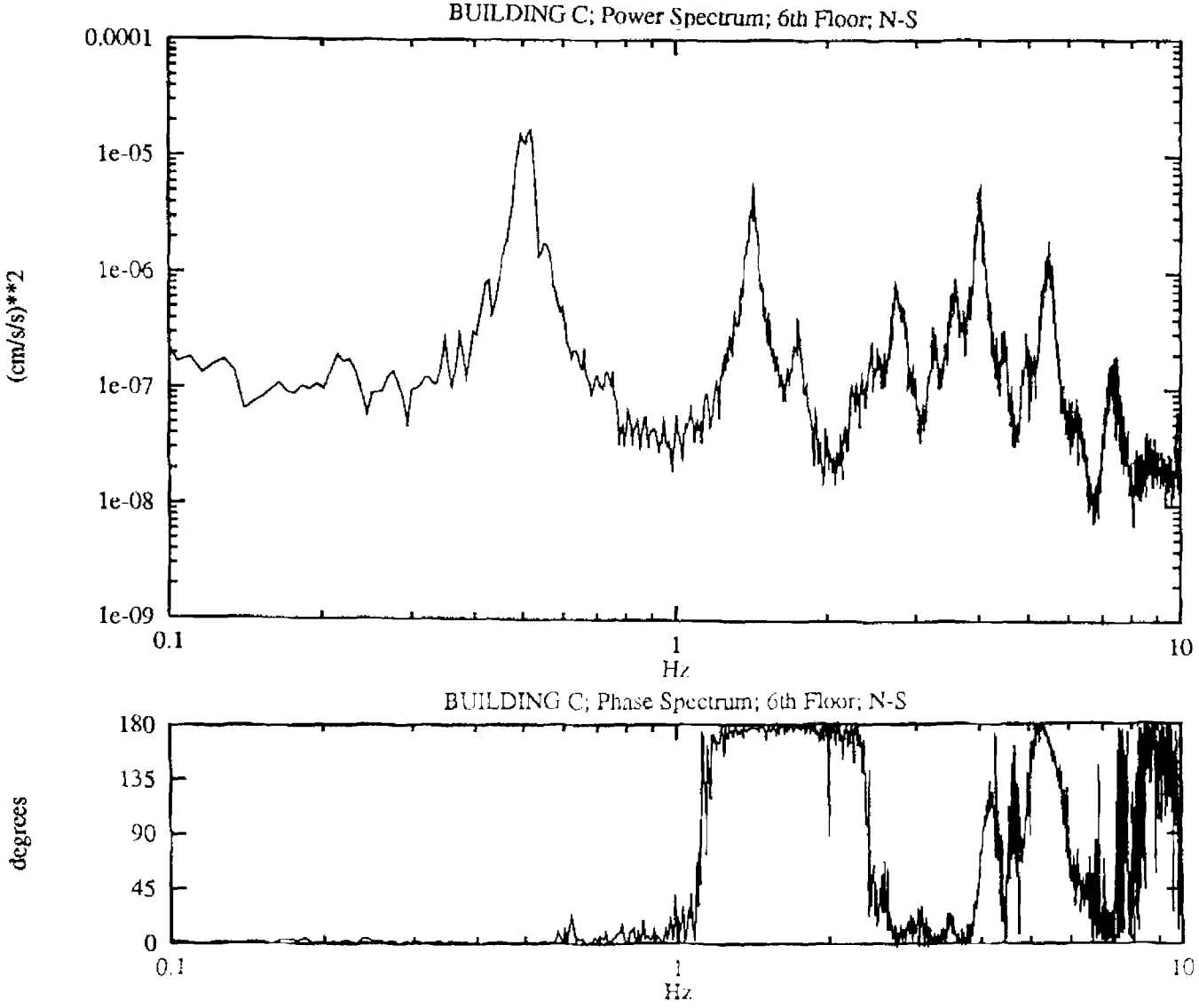
Hz-data		Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2502		0.2512	2.2667e-04	2.2942e-04	2	2.4
0.2869	T1	0.2868	1.3903e-04	1.3904e-04	180	1.6
0.3601		0.3581	8.7529e-05	9.1955e-05	1	1.0
0.5005		0.5002	3.1643e-06	3.1674e-06	71	0.7
0.8301	T2	0.8288	3.1950e-04	3.2389e-04	179	0.9
0.9277	T3	0.9304	5.3538e-06	5.5768e-06	170	1.2
1.0010		1.0000	2.1989e-06	2.2259e-06	8	0.5
1.2451		1.2470	2.1100e-05	2.1651e-05	91	0.8
1.4099	T4	1.4127	3.5441e-05	3.6925e-05	177	0.7
1.7639		1.7650	4.5221e-05	4.5329e-05	5	0.8
1.9531	T5	1.9521	7.9733e-05	8.0347e-05	175	0.5
2.1667		2.1654	5.9476e-06	5.9903e-06	3	0.5
2.5757	T6	2.5757	1.9286e-04	1.9264e-04	175	0.3
2.8259		2.8251	2.2969e-05	2.3007e-05	8	0.5
3.3020	T7	3.3027	2.1444e-04	2.1553e-04	174	0.4
3.8208		3.8222	7.8104e-05	7.8201e-05	1	0.6
4.1382		4.1370	9.6194e-05	9.5367e-05	159	1.2

RMS-acc	RMS-vel	RMS-dsp
1.850e-01	2.066e-02	1.179e-02

FIGURE B-19

BUILDING C - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 22, 1991
Measurement	Sensor Location	6th Floor, Center
Reference	Sensor Location	28th Floor, Center
	Sensor Direction	North-South



Peaks In G6.28n

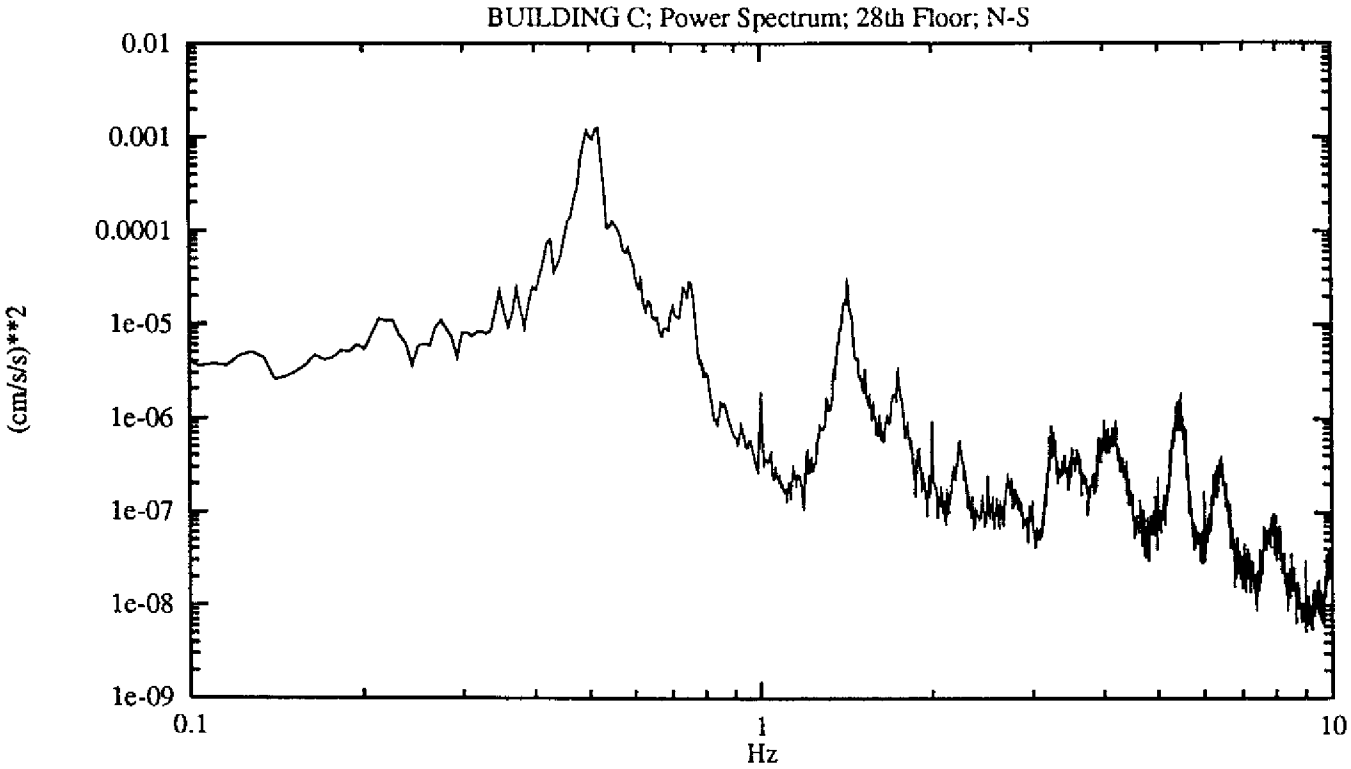
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.5188 N1	0.5162	1.6565e-05	1.7408e-05	0	1.4
1.4221 T2	1.4241	5.4219e-06	5.5581e-06	177	0.6
1.5259 N2	1.5257	4.5955e-07	4.5914e-07	178	0.7
2.7344 T4	2.7336	8.6550e-07	8.6427e-07	3	1.8
4.0222 T5	4.0209	5.4778e-06	5.4985e-06	78	0.5
5.4749 N3	5.4720	1.8704e-06	1.9670e-06	162	0.3

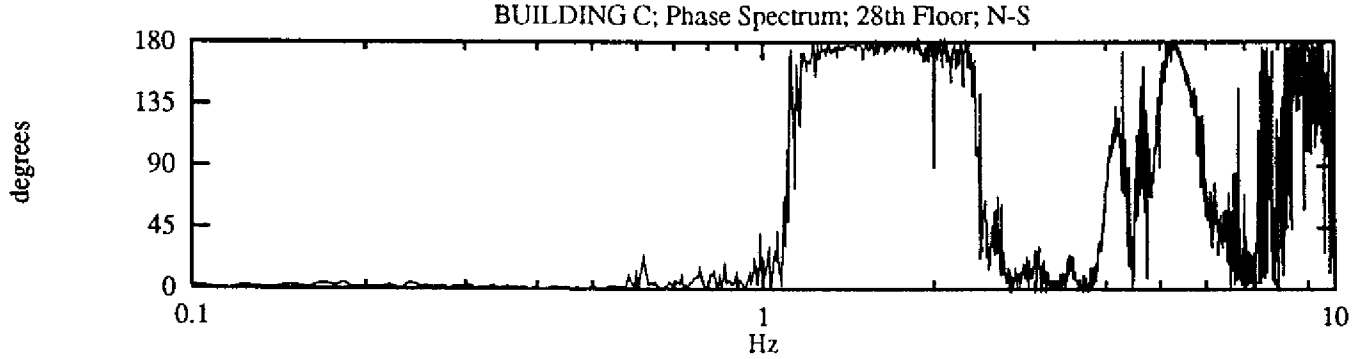
RMS-acc	RMS-vel	RMS-dsp
2.288e-02	4.878e-03	3.068e-03

FIGURE B-20

BUILDING C - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 22, 1991
Measurement	Sensor Location	28th Floor, Center
Reference	Sensor Location	6th Floor, Center
	Sensor Direction	North-South





Peaks In G28.6n

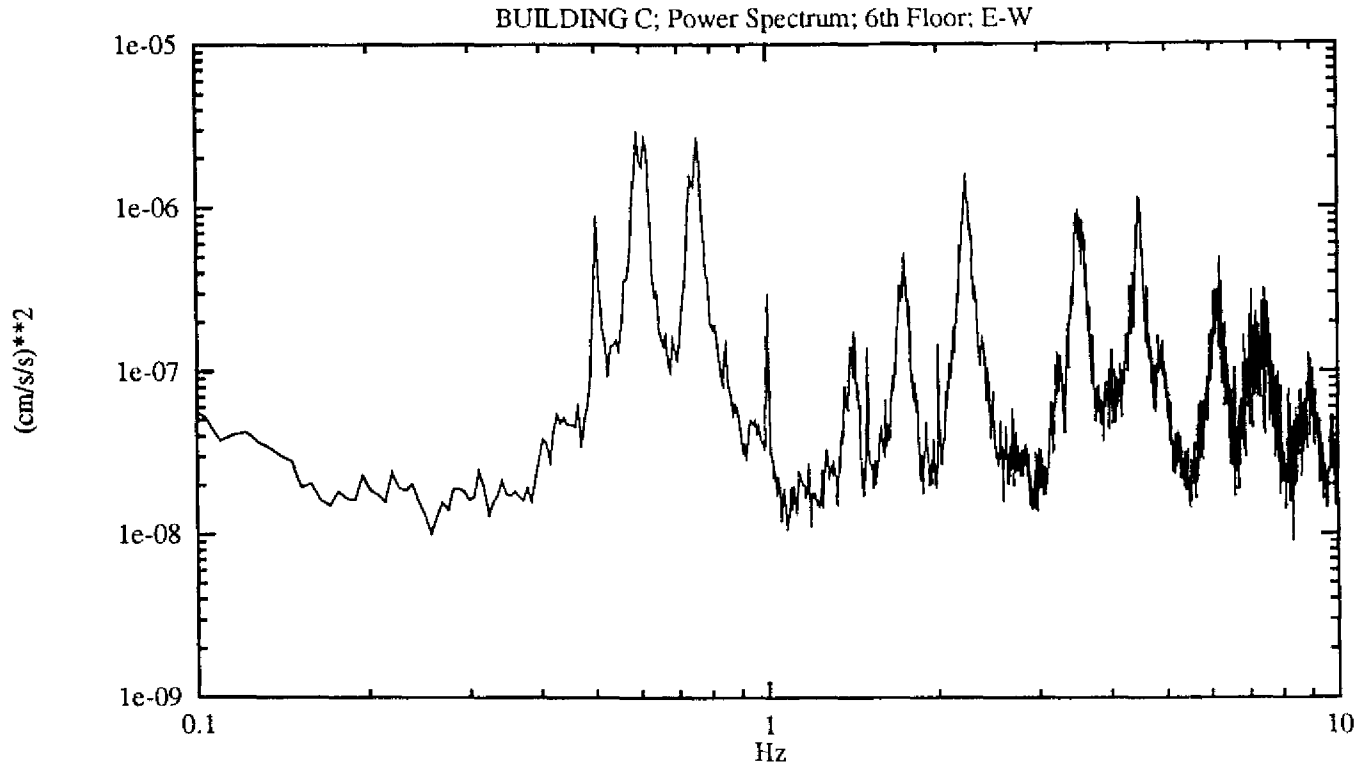
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.5188 N1	0.5162	1.2670e-03	1.3330e-03	0	1.4
0.7507 T1	0.7533	2.8378e-05	2.9229e-05	4	1.6
1.4221 T2	1.4241	2.7987e-05	2.8670e-05	177	0.7
1.7456	1.7453	3.0109e-06	3.0138e-06	177	1.1
5.4749 N3	5.4728	1.6624e-06	1.6987e-06	162	0.3

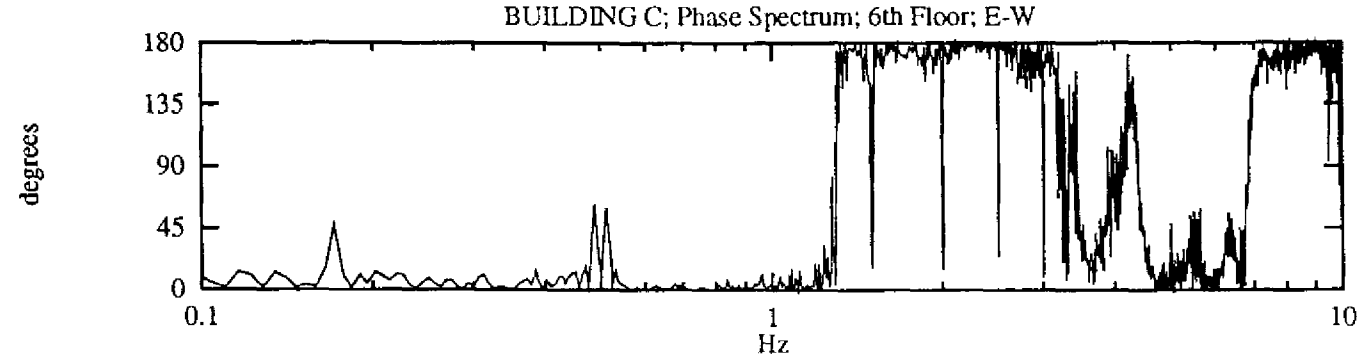
RMS-acc	RMS-vel	RMS-dsp
1.080e-01	3.532e-02	1.527e-02

FIGURE B-21

BUILDING C - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 22, 1991
Measurement	Sensor Location	6th Floor, Center
Reference	Sensor Location	28th Floor, Center
	Sensor Direction	East-West





Peaks In G6.28e

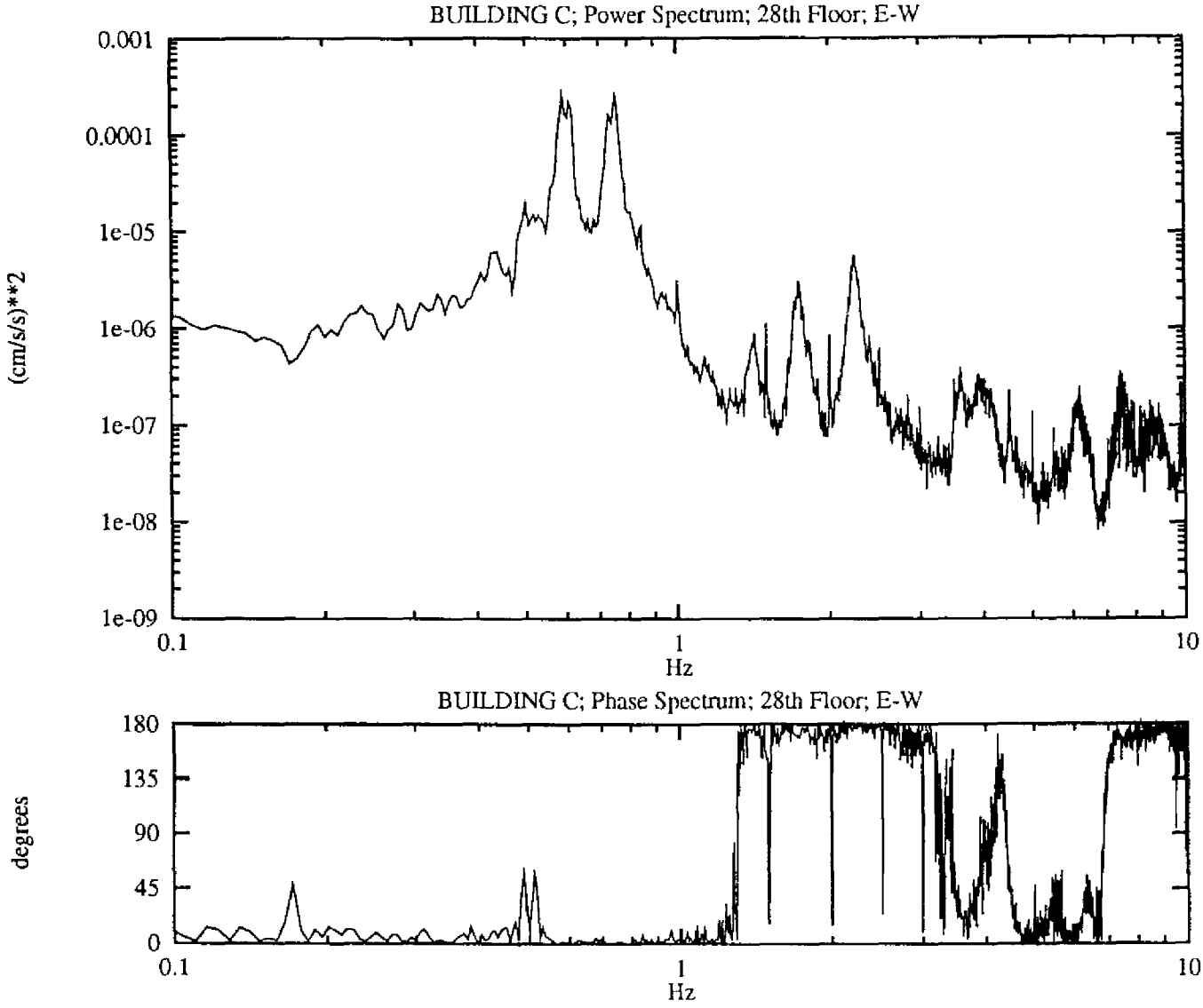
Hz-data	Hz-flt	Amp^2-data	Amp^2-flt	Phase	Damping %
0.5005	0.5004	8.9375e-07	8.9360e-07	10	0.9
0.5920	0.5924	2.9490e-06	2.9523e-06	1	3.3
0.7568	0.7559	2.5533e-06	2.5667e-06	2	1.3
0.8118	0.8102	1.9406e-07	1.9581e-07	4	2.1
1.0010	1.0006	2.9809e-07	2.9895e-07	7	0.4
1.4221	1.4238	1.7510e-07	1.7881e-07	171	2.6
1.7456	1.7435	4.9491e-07	5.0385e-07	175	1.8
2.2400	2.2384	1.5507e-06	1.5581e-06	177	1.3
3.5156	3.5162	9.6818e-07	9.6858e-07	38	1.1
3.5828	3.5825	8.3206e-07	8.3447e-07	19	1.5
4.4800	4.4825	1.1470e-06	1.1474e-06	45	1.3

RMS-acc	RMS-vel	RMS-dsp
1.660e-02	2.044e-03	1.168e-03

FIGURE B-22

BUILDING C - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 22, 1991
Measurement	Sensor Location	28th Floor, Center
Reference	Sensor Location	6th Floor, Center
	Sensor Direction	East-West



Peaks In G28.6e

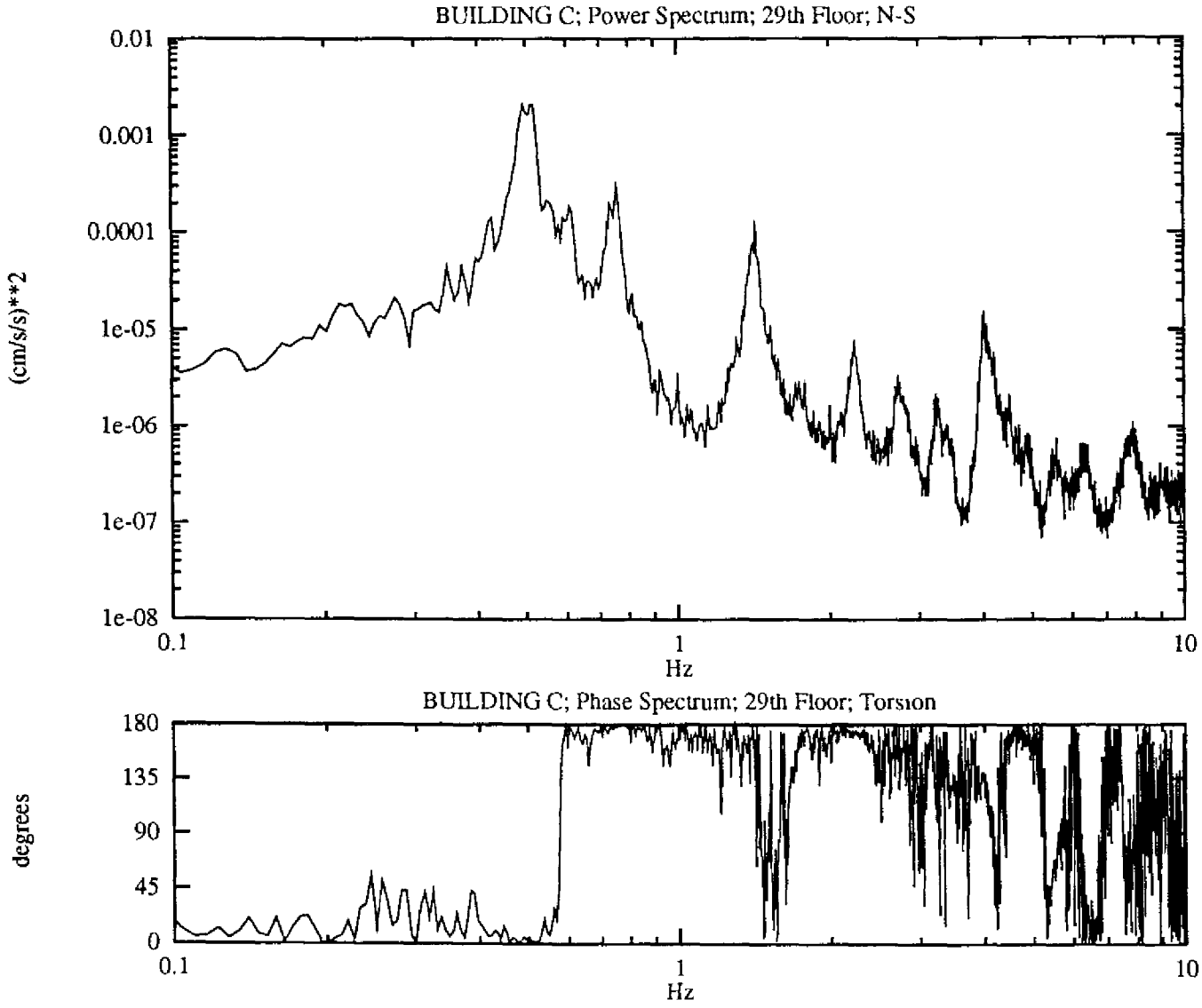
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.5920 E1	0.5922	2.6638e-04	2.6637e-04	1	2.2
0.7568 T1	0.7562	2.5561e-04	2.5630e-04	2	1.2
1.0010	1.0012	3.1784e-06	3.1851e-06	7	0.7
1.4221 T2	1.4219	8.8905e-07	8.8848e-07	171	1.3
1.5015 E2	1.5007	1.1213e-06	1.1288e-06	16	0.2
1.7395 E3	1.7423	3.0454e-06	3.1516e-06	172	1.3
2.2400 E4	2.2425	5.6826e-06	5.6941e-06	177	1.3

RMS-acc	RMS-vel	RMS-dsp
6.280e-02	1.582e-02	6.321e-03

FIGURE B-23

BUILDING C - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 22, 1991
Measurement	Sensor Location	29th Floor, West
Reference	Sensor Location	29th Floor, East
	Sensor Direction	North-South



Peaks In G29w.29en

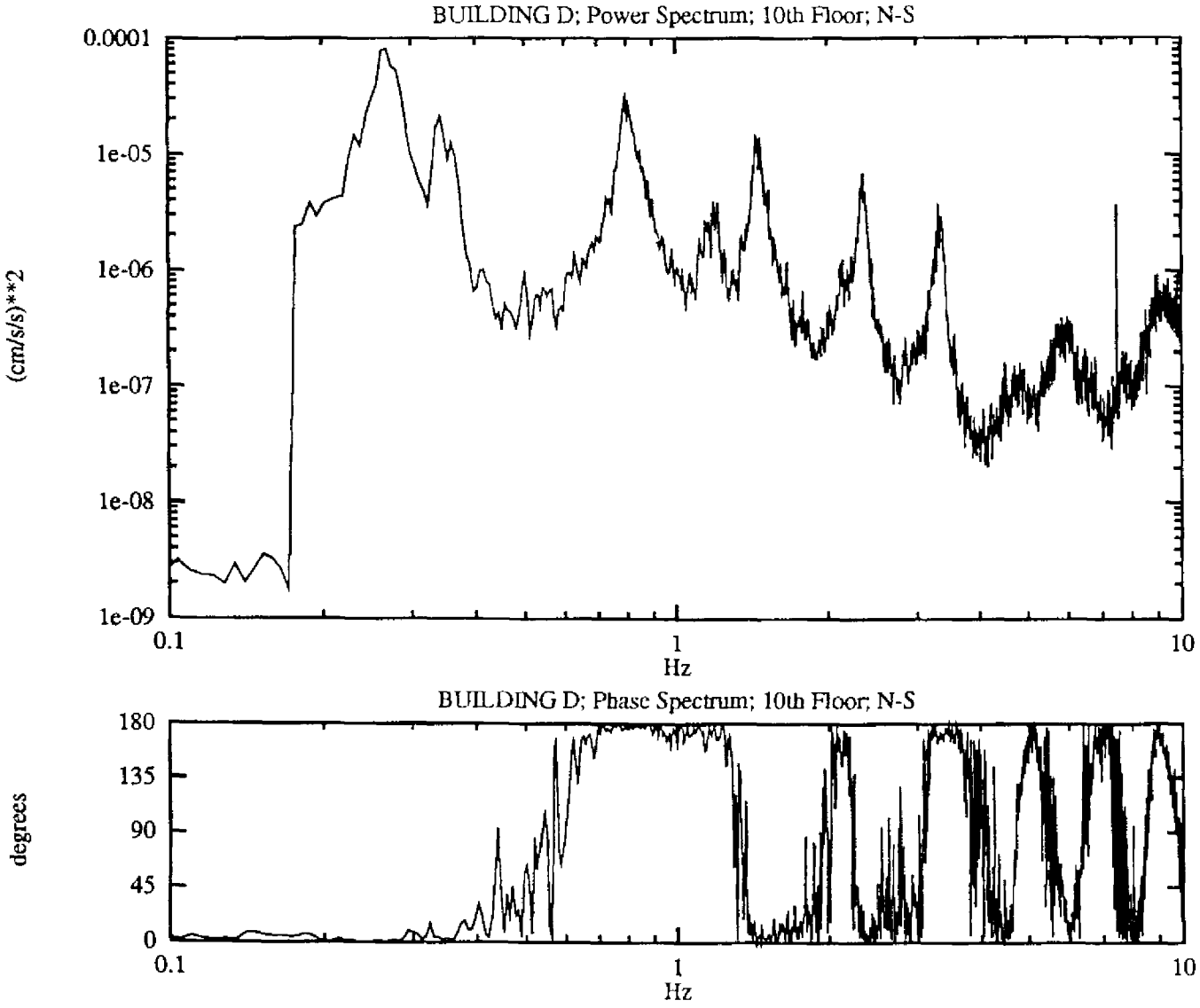
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.5188 N1	0.5158	2.1116e-03	2.2473e-03	2	1.5
0.7568 T1	0.7568	2.9631e-04	2.9635e-04	179	1.3
1.4221 T2	1.4240	1.2100e-04	1.2374e-04	178	0.7
2.2400 T3	2.2375	7.7551e-06	7.8529e-06	175	1.5
2.7344 T4	2.7320	3.2104e-06	3.2857e-06	178	1.3
3.2349	3.2347	2.2786e-06	2.2352e-06	94	0.2
4.0222 T5	4.0211	1.4396e-05	1.4484e-05	139	0.6

RMS-acc	RMS-vel	RMS-dsp
1.549e-01	4.747e-02	1.890e-02

FIGURE B-24

BUILDING D - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 22, 1991
Measurement	Sensor Location	10th Floor, Center
Reference	Sensor Location	51st Floor, Center
	Sensor Direction	North-South



Peaks In G10.51n

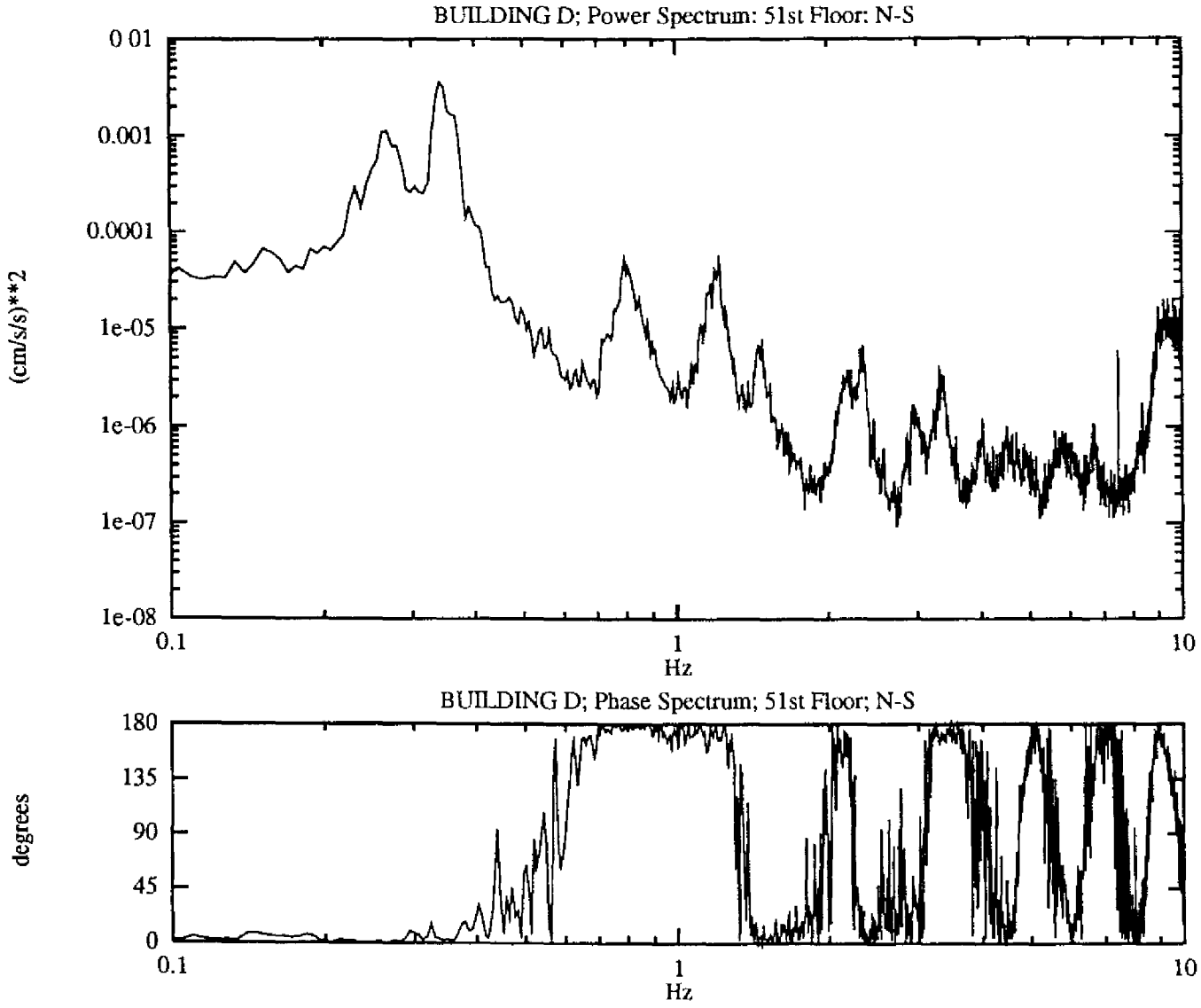
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2686 N1	0.2660	8.1053e-05	8.3365e-05	1	5.7
0.3418 T1	0.3413	2.0928e-05	2.0962e-05	2	2.8
0.7935 N2	0.7926	3.1094e-05	3.1233e-05	176	1.2
1.1902 T2	1.1902	3.8834e-06	3.8855e-06	178	1.2
1.4465 N3	1.4491	1.4324e-05	1.4625e-05	4	1.1
2.3499 T3	2.3482	6.8481e-06	6.9141e-06	6	0.4
3.3142 T4	3.3156	3.7621e-06	3.7849e-06	171	0.3

RMS-acc	RMS-vel	RMS-dsp
4.324e-02	1.525e-02	8.904e-03

FIGURE B-25

BUILDING D - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 22, 1991
Measurement	Sensor Location	51st Floor, Center
Reference	Sensor Location	10th Floor, Center
	Sensor Direction	North-South



Peaks In G51.10n

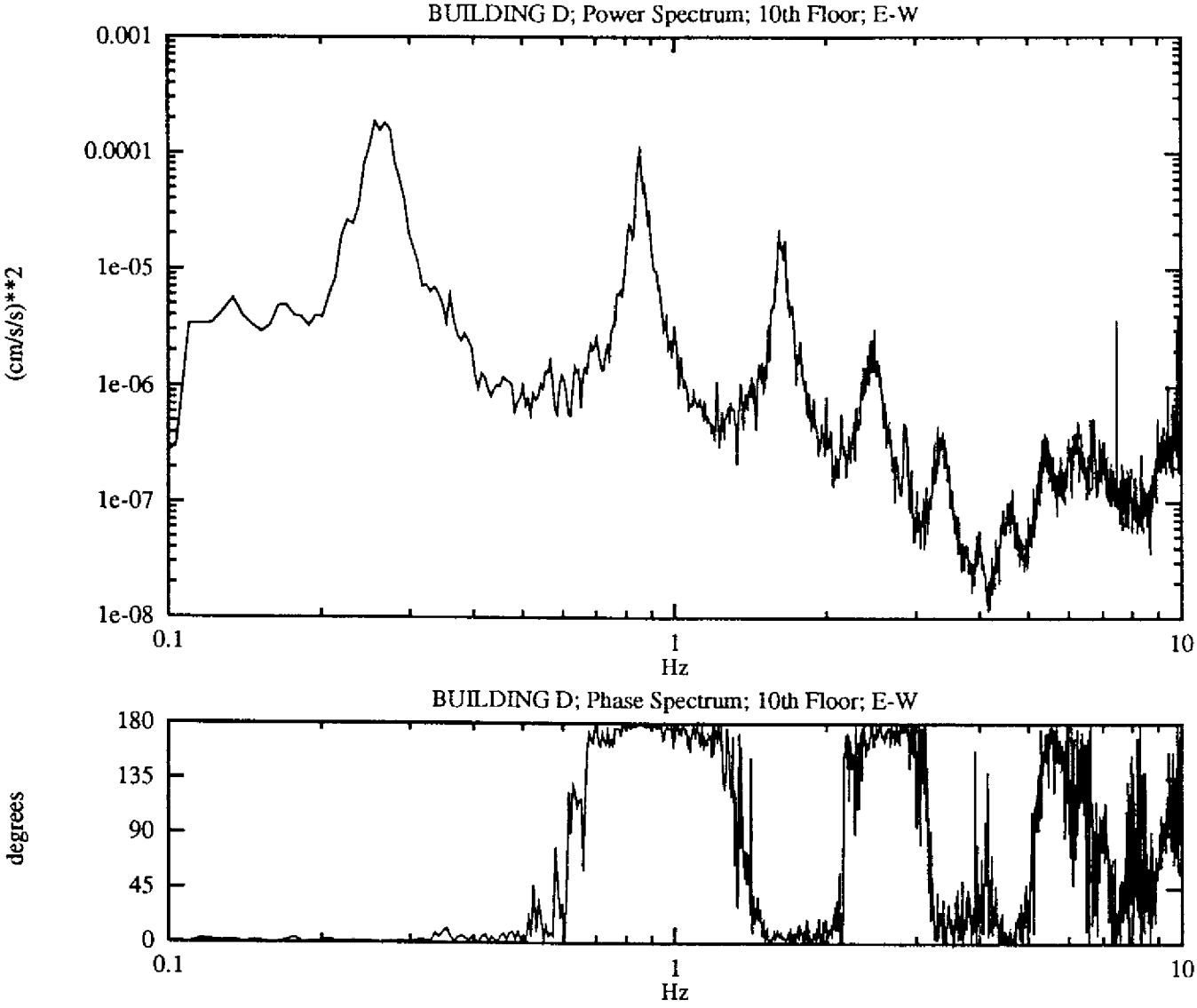
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2686 N1	0.2663	1.1406e-03	1.1693e-03	1	3.8
0.3418 T1	0.3434	3.5929e-03	3.6454e-03	2	2.7
0.7935 N2	0.7929	5.2582e-05	5.2661e-05	176	1.2
1.2146 T3	1.2148	5.7150e-05	5.7101e-05	167	0.7
1.4771 N3	1.4766	7.9147e-06	7.9274e-06	6	0.6
2.1423	2.1427	3.4101e-06	3.4422e-06	169	3.3
2.3499 T3	2.3491	6.8024e-06	6.7651e-06	6	0.6
3.3142 T4	3.3167	3.8288e-06	3.9488e-06	171	0.2

RMS-acc	RMS-vel	RMS-dsp
1.811e-01	8.682e-02	5.197e-02

FIGURE B-26

BUILDING D - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 22, 1991
Measurement	Sensor Location	10th Floor, Center
Reference	Sensor Location	51st Floor, Center
	Sensor Direction	East-West



Peaks In G10.51e

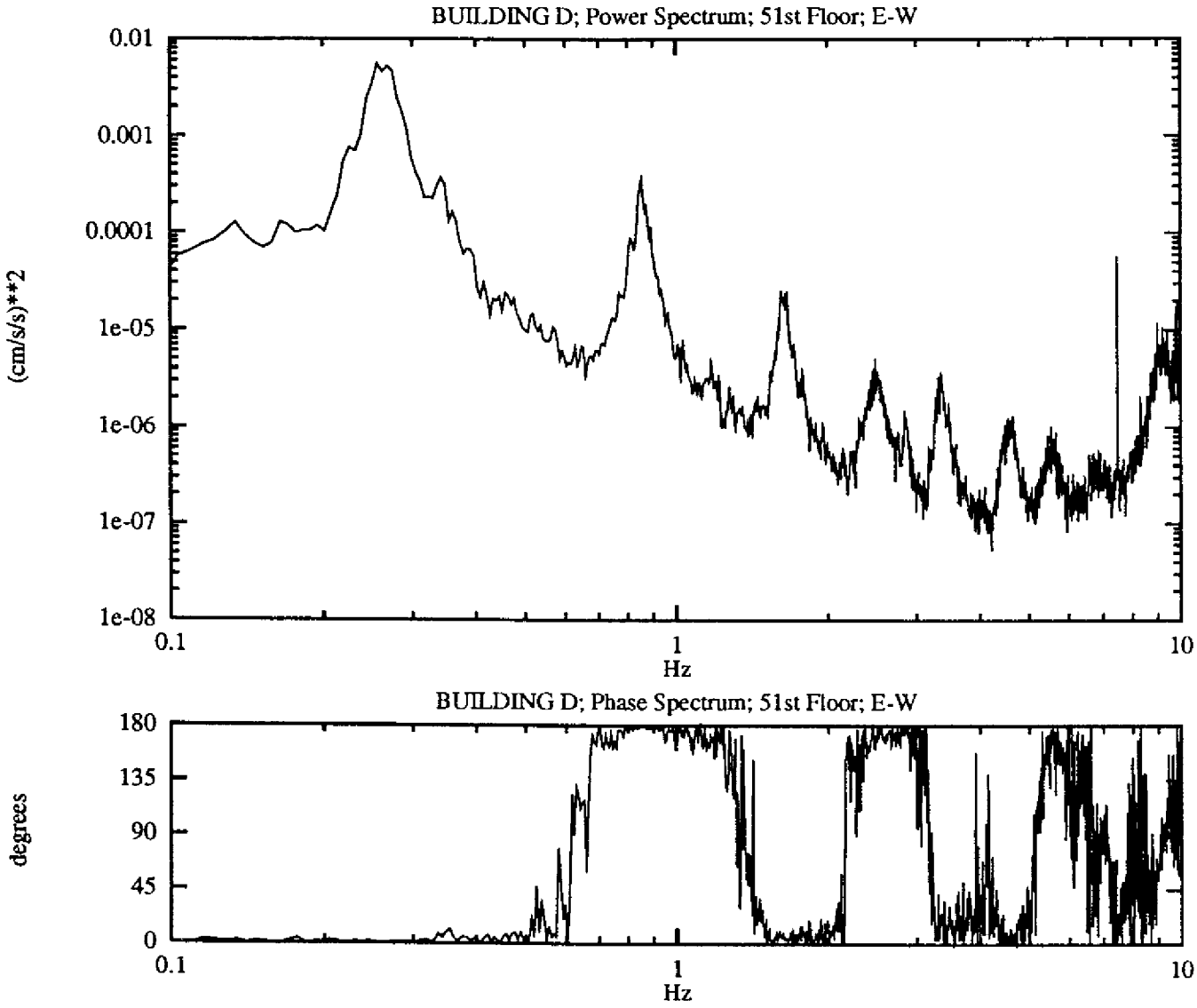
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2563 E1	0.2575	1.8771e-04	1.8965e-04	1	6.0
0.8545 E2	0.8531	1.0513e-04	1.0663e-04	179	1.0
1.6174 E3	1.6161	2.1657e-05	2.1815e-05	6	1.6
2.4902 E4	2.4897	3.0756e-06	3.0994e-06	170	0.3
3.3203 T4	3.3189	4.6188e-07	4.5821e-07	12	0.3

RMS-acc	RMS-vel	RMS-dsp
5.490e-02	2.386e-02	1.531e-02

FIGURE B-27

BUILDING D - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 22, 1991
Measurement	Sensor Location	51st Floor, Center
Reference	Sensor Location	10th Floor, Center
	Sensor Direction	East-West



Peaks In G51.10e

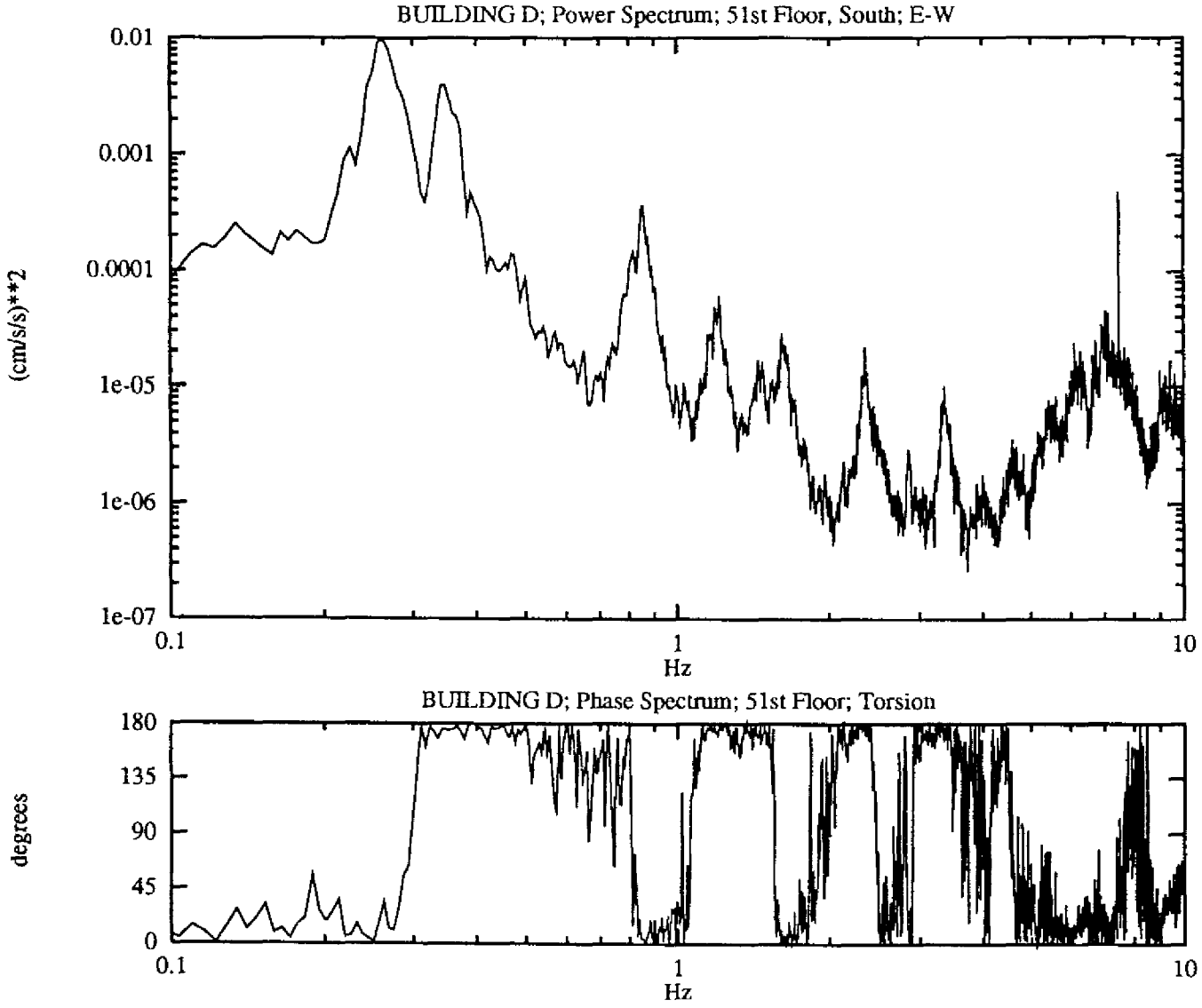
Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2563 E1	0.2575	5.6205e-03	5.6789e-03	1	5.9
0.3418 T1	0.3419	3.6986e-04	3.6991e-04	5	2.4
0.8545 E2	0.8531	3.6434e-04	3.6907e-04	179	1.0
1.6174 E3	1.6169	2.5655e-05	2.5660e-05	6	2.4
2.4902 E4	2.4896	5.1716e-06	5.1856e-06	170	0.3
3.3630 T4	3.3633	3.3579e-06	3.3528e-06	24	1.0

RMS-acc	RMS-vel	RMS-dsp
2.166e-01	1.259e-01	8.251e-02

FIGURE B-28

BUILDING D - Power and Phase Spectra of Measured Acceleration

Measurement	Date	February 22, 1991
Measurement	Sensor Location	51st Floor, South
Reference	Sensor Location	51st Floor, North
	Sensor Direction	East-West



Peaks In G51s.51ne

Hz-data	Hz-fit	Amp^2-data	Amp^2-fit	Phase	Damping %
0.2625 E1	0.2606	9.6744e-03	9.7719e-03	34	5.0
0.3479 T1	0.3450	3.9176e-03	4.0276e-03	175	4.1
0.8545 E2	0.8516	3.6310e-04	3.7861e-04	3	1.3
1.2146 T2	1.2143	6.0564e-05	6.0499e-05	179	0.7
1.6174 E3	1.6160	2.6603e-05	2.6822e-05	1	1.6
2.3499 T3	2.3489	2.1690e-05	2.1636e-05	176	0.4
3.3569 T4	3.3577	1.0105e-05	1.0192e-05	177	0.5

RMS-acc	RMS-vel	RMS-dsp
3.221e-01	1.743e-01	1.117e-01